Fachbereich Mathematik
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1st Exercise Sheet Analysis II (engl.)
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(G1.1)

Compute the following integrals.

1. $\displaystyle\int_0^{\sqrt{\pi}} 2x \cdot \cos(x^2)dx$

2. $\displaystyle\int_1^e \frac{x}{x+1}dx$

3. $\displaystyle\int_0^{1/2} \frac{1}{x^2-1}dx$

(G1.2)

1. Give the example of a differentiable function $f : \mathbb{R} \to \mathbb{R}$ which is not periodic but f' is periodic.

   Hint. One may start by considering the function $\sin(x)$.

2. Suppose that $g : [0,1] \to \mathbb{R}$ is a jump continuous function which is continuous at 0 and $g(0) = 0$. Prove that
$$\lim_{x \to 0^+} \frac{1}{x} \cdot \int_0^x g(t)dt = 0.$$

(G1.3)

1. Define the function $F : \mathbb{R} \to \mathbb{R}$ by $F(x) = \int_0^{x^2} \sin(1+t)dt$, $x \in \mathbb{R}$. Explain why F is differentiable and compute F'.

2. Define the function $F : [0,1] \to \mathbb{R}$ by $F(x) = \int_0^x 1 + \sin(\sin(t))dt$, $x \in [0,1]$. Prove that F is strictly increasing and compute the number $(F^{-1})'(0)$.

   Hint. Do not try to compute F^{-1}.