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Introduction to Mathematical Logic

SS 2010, Exercise Sheet #8

EXERCISE 29:

a) Define a term F in three parameters such that
for all x, y, z it holds $F(\emptyset, x, y) = x$ and, in case $z \neq \emptyset$, $F(z, x, y) = y$.

b) Prove formally: $y = \emptyset \Leftrightarrow \forall z : z \notin y$.

c) Use "$\in, \vee, \neg, \exists$" and the abbreviations from the lecture to formally express the predicate $P(f)$ that "all elements of f are ordered pairs".

EXERCISE 30:

a) Cook up a formal predicate $P(x)$ expressing that "x has precisely one element".

b) Let $(X_i)_{i\in I}$ be a family of sets such that, for every $i \in I$, X_i has precisely one element.
Prove, avoiding the axiom of choice: $\prod_{i\in I} X_i \neq \emptyset$.

Recall from Exercise 19e) the following two claims:

i) To every injective mapping $f : A \to B$,
there exists a (necessarily surjective) mapping $g : B \to A$ with $g \circ f = \mathrm{id}_A$.

ii) To every surjective mapping $g : B \to A$,
there exists a (necessarily injective) mapping $f : A \to B$ with $g \circ f = \mathrm{id}_A$.

EXERCISE 31:

a) Express both i) and ii) formally.
(You may introduce and use abbreviations.)

b) Which claim of i),ii) can be proven without the axiom of choice?
Give such a proof!

c) Prove that the other claim implies the axiom of choice.
Hint: consider $g : (i, x) \mapsto i$.