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Introduction to Mathematical Logic

SS 2010, Exercise Sheet #2

EXERCISE 11:

a) Give three distinct examples of ordinals.
Prove that they are indeed ordinals.

b) Let α denote an ordinal. Prove that $\alpha^+ := \alpha \cup \{\alpha\}$ is again an ordinal.

c) Give an example of a *limit ordinal*, i.e. one which is *not* of the form α^+ nor the empty set.
Again, prove that it is an ordinal.

d) Similarly to Exercise 9, draw two ordered sets:
one isomorphic to the ordinal $\omega + 2$ and one isomorphic to $\omega \times 2$.

EXERCISE 12:
Which of the Peano axioms are satisfied by the structure $\big(\omega \times 2, \alpha \mapsto \alpha^+\big)$,
which ones are violated?

EXERCISE 13:
Prove that $\big\{\alpha : \alpha \text{ ordinal}\big\}$ is not a set.

EXERCISE 14:
Abbreviate $1 := \emptyset^+$ and $2 := 1^+$.
Prove "$1+1=2$" by stating explicitly an isomorphism between ordered sets $1 \uplus 1$ and 2.
(This knowledge will come handy to impress your non-mathematician friends or parents…)

EXERCISE 15:
Show that ordinal arithmetic is in general not commutative; specifically, prove:

a) $1 + \omega = \omega < \omega + 1$

b) $2 \times \omega = \omega < \omega \times 2$.