



# Höhere Mathematik 1

## 3. Übung, Lösungsvorschlag

### Gruppenübungen

#### Aufgabe G7

a)

$$\log_{10} \sqrt[3]{\left(\frac{a^2}{b^2}\right)} = \frac{1}{3} \log_{10} \left(\frac{a^2}{b^2}\right) = \frac{1}{3} \log_{10}(a^2) - \frac{1}{3} \log_{10}(b^2) = \frac{2}{3} \log_{10} a - \frac{2}{3} \log_{10} b$$

b)

$$\begin{aligned} 3 \left( \ln 2 - 2 \ln x - \frac{1}{2} \ln y \right) &= 3(\ln 2 - \ln(x^2) - \ln \sqrt{y}) = 3 \left( \ln \left( \frac{2}{x^2} \right) - \ln \sqrt{y} \right) \\ &= 3 \ln \left( \frac{2}{x^2 \sqrt{y}} \right) = \ln \left( \frac{2^3}{(x^2)^3 (\sqrt{y})^3} \right) = \ln \left( \frac{8}{x^6 \sqrt{y^3}} \right) \end{aligned}$$

c)

$$\begin{aligned} \left( \frac{1}{12} \right)^x &= 5^6 \\ \log \left( \left( \frac{1}{12} \right)^x \right) &= \log 5^6 \\ x \log \left( \frac{1}{12} \right) &= 6 \log 5 \\ x (\log 1 - \log 12) &= 6 \log 5 \\ -x \log 12 &= 6 \log 5 \\ x &= -6 \frac{\log 5}{\log 12} \end{aligned}$$

#### Aufgabe G8

a)

$$\begin{aligned} 3^{-2x} &= 4^{5x-3} \\ \ln(3^{-2x}) &= \ln(4^{5x-3}) \\ -2x \ln 3 &= (5x-3) \ln 4 \\ x(-2 \ln 3 - 5 \ln 4) &= -3 \ln 4 \\ x &= \frac{3 \ln 4}{2 \ln 3 + 5 \ln 4} \end{aligned}$$

b)

$$\begin{aligned}5^{x-1} + 5^{x+1} &= 6^x \\ \frac{1}{5} \cdot 5^x + 5 \cdot 5^x &= 6^x \\ \frac{26}{5} \cdot 5^x &= 6^x \\ \ln\left(\frac{26}{5} \cdot 5^x\right) &= \ln(6^x) \\ \ln \frac{26}{5} + \ln(5^x) &= \ln(6^x) \\ \ln \frac{26}{5} + x \ln 5 &= x \ln 6 \\ \ln \frac{26}{5} &= x(\ln 6 - \ln 5) \\ \frac{\ln \frac{26}{5}}{\ln 6 - \ln 5} &= x\end{aligned}$$

c)

$$\begin{aligned}e^x &= 5e^{-x} + 4 \\ (e^x)^2 &= 5 + 4e^x \\ (e^x)^2 - 4(e^x) - 5 &= 0 \\ e^x &= 2 \pm \sqrt{4 + 5} = 2 \pm 3 \\ e^x &= -1 \quad \text{oder} \quad e^x = 5\end{aligned}$$

Die Gleichung  $e^x = -1$  hat keine Lösung, also bleibt nur die Lösung  $x = \ln 5$ .

**Aufgabe G9** Die Nullstellen der auftretenden Nenner können keineswegs Lösungen sein, also  $x \notin \{0, 2, 3\}$ . Unter dieser Annahme gilt

$$\begin{aligned}(5+x)(x-2)x - 2x(3-x)x &= (8-3x)(3-x)(x-2) \\ x^3 + 3x^2 - 10x + 2x^3 - 6x^2 &= -8x^2 + 40x - 48 + 3x^3 - 15x^2 + 18x \\ 3x^2 - 6x^2 + 8x^2 + 15x^2 - 10x - 40x - 18x + 48 &= 0 \\ 20x^2 - 68x + 48 &= 0 \\ x^2 - \frac{17}{5}x + \frac{12}{5} &= 0 \\ x &= \frac{17}{10} \pm \sqrt{\frac{289 - 12 \cdot 20}{100}} \\ x &= \frac{17}{10} \pm \frac{7}{10} \\ x &= 1 \quad \text{oder} \quad x = \frac{12}{5}\end{aligned}$$

## Hausübungen

### Aufgabe H7

a)

$$\frac{x}{\sqrt[4]{x^3}} = \frac{x}{x^{\frac{3}{4}}} = x \cdot x^{-\frac{3}{4}} = x^{1-\frac{3}{4}} = x^{\frac{1}{4}}$$

b)

$$\left(\sqrt[5]{a^2b^4}\right)^3 \cdot \left(\sqrt[5]{a^2b^9}\right)^2 = \left(a^{\frac{2}{5}}b^{\frac{4}{5}}\right)^3 \cdot \left(a^{\frac{2}{5}}b^{\frac{9}{5}}\right)^2 = a^{\frac{6}{5}}b^{\frac{12}{5}}a^{\frac{4}{5}}b^{\frac{18}{5}} = a^2b^6$$

c)

$$\begin{aligned} \left(\sqrt[8]{\frac{x^5}{y^4}} \cdot \sqrt[12]{\frac{x}{y}}\right) : \sqrt[6]{\frac{x^4}{y^3}} &= \left(\frac{x^{\frac{5}{8}}}{y^{\frac{4}{8}}} \cdot \frac{x^{\frac{1}{12}}}{y^{\frac{1}{12}}}\right) \cdot \left(\frac{x^{\frac{4}{6}}}{y^{\frac{3}{6}}}\right)^{-1} = \frac{x^{\frac{5}{8} + \frac{1}{12} - \frac{2}{3}}}{y^{\frac{1}{2} + \frac{1}{12} - \frac{1}{2}}} \\ &= \frac{x^{\frac{1}{24}}}{y^{\frac{1}{12}}} = \frac{x^{\frac{1}{24}}y^{\frac{11}{12}}}{y^{\frac{1+11}{12}}} = \frac{x^{\frac{1}{24}}y^{\frac{11}{12}}}{y} \end{aligned}$$

### Aufgabe H8

a)

$$\begin{aligned} 2^{(x^2)} &= (2^x)^2 \\ \ln(2^{(x^2)}) &= \ln(2^{2x}) \\ x^2 \cdot \ln 2 &= 2x \cdot \ln 2 \\ x^2 &= 2x \\ x &= 0 \quad \text{oder} \quad x = 2 \end{aligned}$$

b)

$$\begin{aligned} 2^{2x} - 2^{x+1} &= 2^{3+x} - 9 \\ 2^{2x} - 2 \cdot 2^x &= 8 \cdot 2^x - 9 \\ (2x)^2 - 10 \cdot 2^x + 9 &= 0 \\ 2^x &= 5 \pm \sqrt{25 - 9} = 5 \pm 4 \\ 2^x &= 9 \quad \text{oder} \quad 2^x = 1 \\ x &= \frac{\ln 9}{\ln 2} \quad \text{oder} \quad x = 0 \end{aligned}$$

c)

$$\begin{aligned} 3 \ln x &= \frac{9}{4 \ln x} - 3 \\ 3(\ln x)^2 &= \frac{9}{4} - 3 \ln x \\ (\ln x)^2 + \ln x - \frac{3}{4} &= 0 \\ \ln x &= -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{3}{4}} = -\frac{1}{2} \pm 1 \\ \ln x &= -\frac{3}{2} \quad \text{oder} \quad \ln x = \frac{1}{2} \\ x &= e^{-\frac{3}{2}} \quad \text{oder} \quad x = e^{\frac{1}{2}} \end{aligned}$$

### Aufgabe H9

a)  $x \notin \{-2, 1\}$ .

$$\begin{aligned} 2(x-1) &= 3(x+2) \\ 2x-2 &= 3x+6 \\ -8 &= x \end{aligned}$$

b)  $x \notin \{0, 3\}$ .

$$\begin{aligned}2x(x+4) &= (2x+5)(x-3) \\2x^2+8x &= 2x^2-x-15 \\9x &= -15 \\x &= -\frac{5}{3}\end{aligned}$$

c) Mit  $x^2+3x-108 = (x-9)(x+12)$  folgt  $x \notin \{-12, 0, 9\}$  und

$$\begin{aligned}7(x-9)(x+12) - 4x(x-9) &= 3x(x+12) + 84x \\7x^2 + 21x - 7 \cdot 108 - 4x^2 + 36x &= 3x^2 + 36x + 84x \\-63x &= -7 \cdot 108 \\x &= -12\end{aligned}$$

Da  $x \neq -12$  gelten muß, ist die Gleichung unlösbar.