13.IV Exakte DGL

$$f(x,y)\,dx + g(x,y)\,dy = 0$$

mit $f(x,y) = \frac{\partial u}{\partial x}$, $g(x,y) = \frac{\partial u}{\partial y}$

$u(x,y)$ Stammfunktion

Lsg implizit $u(x,y) = C$
Höhenlinie

explizit: $g(x,y) \neq 0 \Rightarrow y = y(x)$

$$y' = -\frac{f(x,y)}{g(x,y}$$

u existiert $\Leftrightarrow \frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$ Schwarz

Ansatz $u(x,y) = F(x,y) + C(y)$, $\frac{\partial F}{\partial x} = f$

$$g = \frac{\partial F}{\partial y} + C'(y)$$

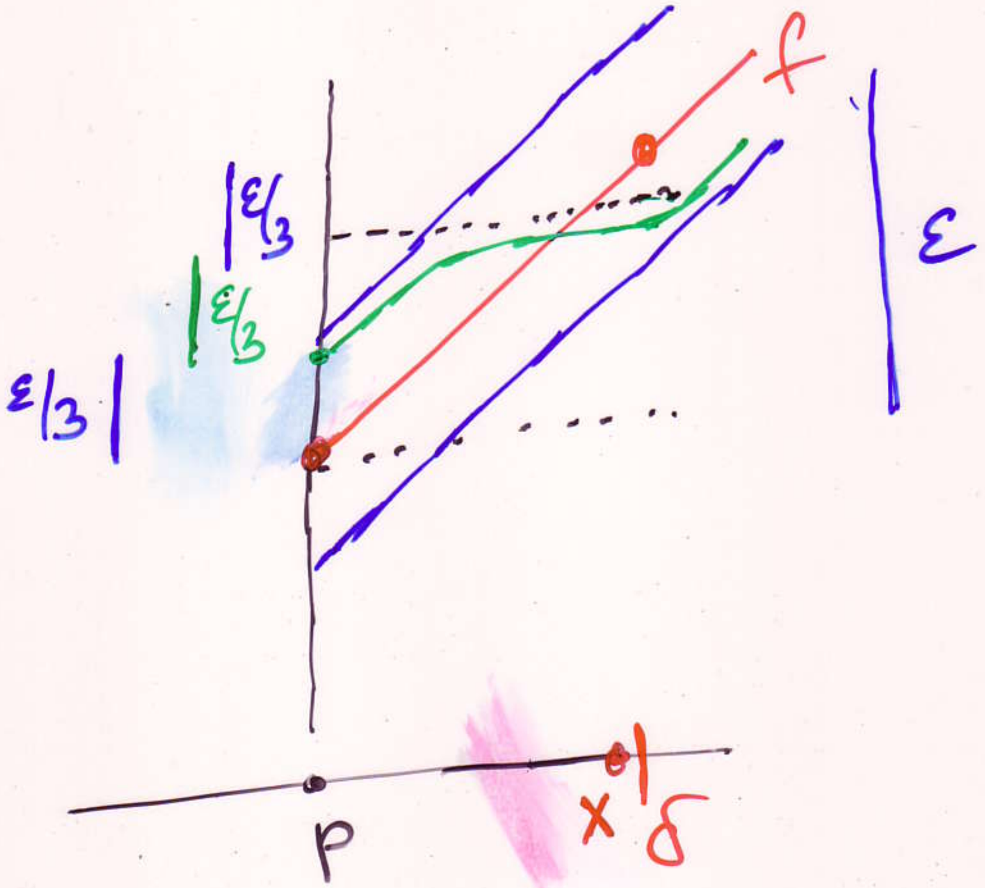
$$\frac{\partial}{\partial x}\left(g - \frac{\partial F}{\partial y}\right) = \frac{\partial g}{\partial x} - \frac{\partial^2 F}{\partial x \partial y}$$

$$= \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} = 0$$

$$\Rightarrow \frac{\partial^2 C}{\partial x \partial y} = 0 \Rightarrow \frac{\partial C}{\partial y} = C'(y) \Rightarrow \text{es gilt } C = C(y)$$

Satz 25.7 $f_n \to f$ glm

f_n stetig $\Longrightarrow$ f stetig



$$\|f_n - f\| \le \varepsilon/3$$

$$\forall x \ |x-p| < \delta \Rightarrow |f_n(x) - f_n(p)| \le \varepsilon/3$$

$$\Rightarrow |f(p) - f(x)|$$

$$\le |f(p) - f_n(p)| + |f_n(p) - f_n(x)| + |f_n(x) - f(x)|$$

$$\le \quad \varepsilon/3 \quad + \quad \varepsilon/3 \quad + \quad \varepsilon/3$$