1.3 III

# Bernoulli'sche DGL

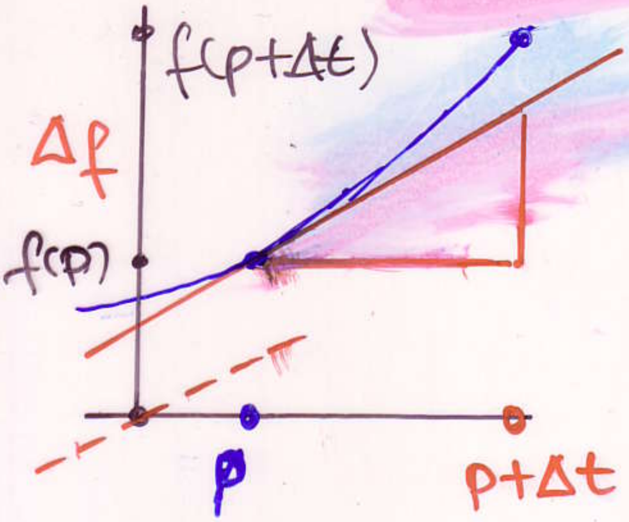
$$y' = p(x)\, y(x) + r(x)\, y(x)^n \qquad n \neq 0,1$$

Lsg. Substitution $z = y^{1-n}$
ergibt lineare DGL für $z$

Bew. $z' = (1-n)\, y^{-n} y'$ Ableiten

$= (1-n)\, y^{-n} (p y + r y^n)$ aus DGL

$= (1-n)\, p \underbrace{y^{1-n}}_{z} + (1-n) r$




$$f(p+\Delta t) = f(p) + \underbrace{a\,\Delta t}_{df(p,\Delta t)} + R(\Delta t)$$

$\frac{R(\Delta t)}{\Delta t} \to 0$ für $\Delta t \to 0$

Differential $\quad \Delta t \mapsto a\,\Delta t$ an $p$

$$a = f'(p) = \frac{\partial f}{\partial t}(p)$$

$dt \mathrel{\hat{=}} \Delta t \qquad df = \frac{\partial f}{\partial t}\, dt$

$$x = x(t), \quad y = y(x) = y(x(t)) = y(t)$$

$$dy(p, dt) = \frac{\partial y}{\partial t}(p) \cdot dt$$

$$= \frac{\partial y}{\partial x}(x(p)) \cdot \underbrace{\frac{\partial x}{\partial t}(p) \cdot dt}_{dx(p,dt)}$$ Kettenregel

$$dy = \frac{\partial y}{\partial x}\, dx$$

$$\frac{\partial y}{\partial x}(x(p)) = \frac{dy(p,dt)}{dx(p,dt)} = \frac{dy}{dx}$$

$dt \neq 0 \qquad$ unabh. von $dt$

Satz $f(x)\,dx = g(y)\,dy$

$\Longleftrightarrow \int f(x)\,dx = \int g(y)\,dy + C$

$f, g$ stetig

Bew. Stammfkt $F, G$

$$\frac{\partial F}{\partial x} = f \qquad \frac{\partial G}{\partial y} = g$$

$$F = G + C$$

$$\Longleftrightarrow \quad \frac{\partial F}{\partial t} = \frac{\partial G}{\partial t}$$

$$\Longleftrightarrow \quad \frac{\partial F}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial G}{\partial y}\frac{\partial y}{\partial t}$$

$$\Longleftrightarrow \quad f\,\frac{\partial x}{\partial t} = g\,\frac{\partial y}{\partial t}$$

$$\Longleftrightarrow \quad f\,\frac{\partial x}{\partial t}\,dt = g\,\frac{\partial y}{\partial t}\,dt \qquad dt \neq 0$$

$$\Longleftrightarrow \quad f\,dx = g\,dy$$

# 1.3 IV Exakte DGL

$z = f(x,y)$ differenzierbar an $(p,q)$

$$f(p+\Delta x, q+\Delta x)$$
$$= f(p,q) + a\Delta x + b\Delta y + R(\Delta x, \Delta y)$$

$$\frac{R(\Delta x, \Delta y)}{\sqrt{\Delta x^2 + \Delta y^2}} \to 0 \text{ für } (\Delta x, \Delta y) \to (0,0)$$

$$a = \frac{\partial f}{\partial x}(p,q)$$
$$b = \frac{\partial f}{\partial y}(p,q)$$

partielle Ableitungen an $(p,q)$

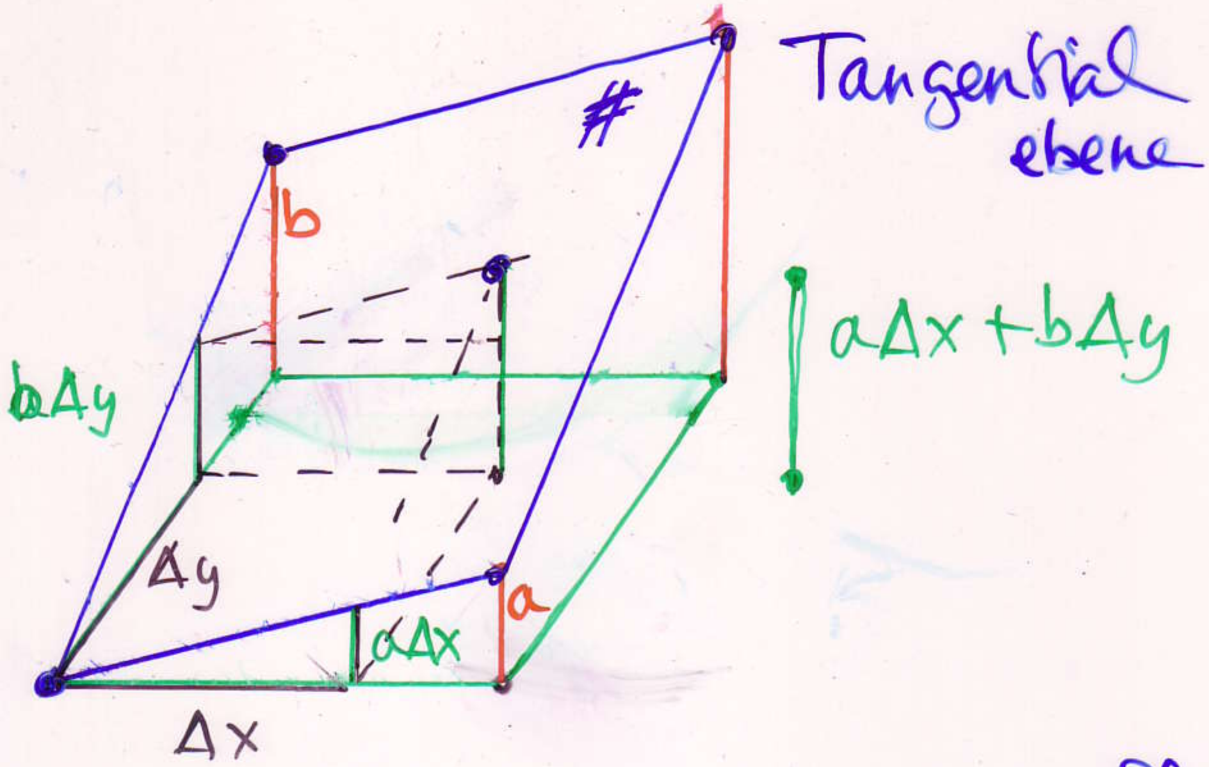
$$\begin{pmatrix} a \\ b \end{pmatrix} = \operatorname{grad} f(p,q)$$

Gradient an $(p,q)$

$$(\Delta x, \Delta y) \mapsto a\Delta x + b\Delta y$$
$$= \left\langle \operatorname{grad} f(p,q) \,\middle|\, \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \right\rangle$$

totales Differential an $(p,q)$

homogen lineare Abbildung





$$\operatorname{grad} f(p,q) = \begin{pmatrix} a \\ b \end{pmatrix} \qquad a = \frac{\partial f}{\partial x}(p,q) \qquad b = \frac{\partial f}{\partial y}(p,q)$$

$$\Delta z = \quad (\Delta x, \Delta y)$$

$$= f(p+\Delta x,\, q+\Delta y) - f(p,q)$$

$$= a\Delta x + b\,\Delta y + R(\Delta x, \Delta y)$$

Kurve $\vec{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ in x-y-Ebene

Tangentialvektor $\frac{\partial \vec{x}}{\partial t} = \begin{pmatrix} \partial x / \partial t \\ \partial y / \partial t \end{pmatrix}$

$$\left\langle \operatorname{grad} f \,\middle|\, \frac{\partial \vec{x}}{\partial t} \right\rangle = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

$$= \frac{\partial f(\vec{x}(t))}{\partial t}$$ Kettenregel

$= 0 \iff f(\vec{x}(t)) = c$ konstant

$\iff \vec{x}(t)$ Höhenlinie

$$\iff \frac{\partial f}{\partial x} \underbrace{\frac{\partial x}{\partial t}\, dt}_{dx} + \frac{\partial f}{\partial y} \underbrace{\frac{\partial y}{\partial t} dt}_{dy} = 0$$

oBdA $n=2,\ p=0 \quad f(x,0)=f(0,y)=0$

$\frac{\partial z}{\partial x}, \frac{\partial z^2}{\partial x \partial y}$ ex lokal, $\frac{\partial z^2}{\partial x \partial y} \to 0 \quad (x,y)\to 0$

$$\frac{1}{t}\,\frac{f(s,t)}{s} = \frac{1}{s}\,\frac{f(s,t)}{t} = \frac{1}{s}\,\frac{\partial z}{\partial y}(s,\tau)$$

$$= \frac{\partial z^2}{\partial x \partial y}(\sigma,\tau) \to 0 \quad \text{für } (s,t)\to 0$$

also $(\sigma,\tau)\to 0$