Fachbereich Mathematik
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13th Homework Sheet Analysis I (engl.)

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(H13.1)

Let $f : \mathbb{R} \to \mathbb{R}$ be bijective, differentiable with inverse function g. We have the following information on f:

(1) $f(0) = 1$ and $f'(0) = 2$.

(2) The derivative of f is bounded: $1 \leq f'(x) \leq 3$ for all $x \in \mathbb{R}$.

Answer the following questions:

(i) Is g everywhere differentiable? For which x does one know the values of $g(x)$ and $g'(x)$?

(ii) Is g' bounded?

(iii) Is f monotone?

(iv) Give an estimate (from below and above) for $f(10)$ via the mean value theorem.

(v) Give an estimate (from below and above) for $g(10)$.

(H13.2)

1. Compute the limits
$$\lim_{x \to 0^+} (1+x)^{1/x}, \quad \lim_{x \to 0^+} x^x.$$

2. Let $f : (0,1) \to \mathbb{R}$ be a convex function and assume that there exists some $x_0 \in (0,1)$ s.t. $f(x) \leq f(x_0)$ for all $x \in (0,1)$. Prove that f is constant.

   Hint. If $x < x_0 < y$ then $x_0 = (1-\lambda)x + \lambda y$ for some $\lambda \in (0,1)$.

(H13.3)

Consider the functions $f(x) = \log(1+x), x > -1$ and $g(x) = \cos(x), x \in \mathbb{R}$.

1. Find the Taylor Polynomials $T_n f$, $T_n g$ near 0 for all $n \in \mathbb{N}$.
2. Give an estimate for the remainders $R_n f(x, 0)$, $R_n g(x, 0)$ for all $x \in (0, 1)$ and for all $n \in \mathbb{N}$.
3. For $x = 0$ determine some $n_0 \in \mathbb{N}$ for which we have that $|R_{n_0} g(1, 0)| \leq 10^{-3}$.