Fachbereich Mathematik
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11th Homework Sheet Analysis I (engl.)

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(H11.1)

Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a continuous function and that $\lim_{x\to-\infty} f(x) = a \in \mathbb{R}$, $\lim_{x\to\infty} f(x) = b \in \mathbb{R}$. Prove that f is uniformly continuous.

(H11.2)

Let the functions $f : (0,1] \to \mathbb{R} : f(x) = \sin(\frac{1}{x})$ and $g : (0,1] \to \mathbb{R} : g(x) = x \cdot e^{\frac{1}{x}}$. Prove the following.

1. The functions f and g are continuous.
2. The functions f and g do not obtain a continuous extension on $[0,1]$ i.e., there is no continuous function $F : [0,1] \to \mathbb{R}$ s.t. $F(x) = f(x)$, for all $x \in [0,1]$ and there is no continuous function $G : [0,1] \to \mathbb{R}$ s.t. $G(x) = g(x)$, for all $x \in [0,1]$.

(H11.3)

Examine whether the following series are convergent, absolutely convergent or divergent.

$$\sum_{n=1}^{\infty} n^2 \cdot \sin(\frac{1}{n^4}), \quad \sum_{n=1}^{\infty} \cos(\frac{1}{n}), \quad \sum_{n=1}^{\infty} (1 - \cos(\frac{1}{n})).$$

Hint. For the third series one may use T11.2-(2).