Fachbereich Mathematik
Prof. Dr. W. Trebels
Dr. V. Gregoriades

TECHNISCHE UNIVERSITÄT DARMSTADT

2009-12-03

8th Homework Sheet Analysis I (engl.)

Winter Term 2009/10

(H8.1)

1. Find the interior and the closure of the following sets:

$$(0,\sqrt{3}],\quad (-\infty,0),\quad \mathbb{R}\setminus\mathbb{Q},\quad \emptyset.$$

2. Which of the previous sets are open?

3. Find a simpler form for the following sets:

$$\bigcup_{n\in\mathbb{N}}[2,(1+\frac{1}{n})^n],\quad \bigcup_{n\in\mathbb{N}}[-n,-\frac{1}{n}),\quad \bigcap_{n\in\mathbb{N}}(-\frac{1}{n},\frac{1}{n}]\quad \bigcap_{n\in\mathbb{N}}(-\infty,-n).$$

(H8.2)

1. Prove Theorem 2.6 Chap. 3 i.e.,

   (a) The sets $\emptyset, \mathbb{R}^n$ are closed subsets of $\mathbb{R}^n$.

   (b) If F_i is a closed subset of $\mathbb{R}^n$ for all $i \in I$ then the intersection $\bigcap_{i\in I} F_i$ is a closed subset of $\mathbb{R}^n$.

   (c) If $F_1,\dots,F_m$ are closed subsets of $\mathbb{R}^n$ then the union $F_1\cup\ldots\cup F_m$ is a closed subset of $\mathbb{R}^n$.

2. Find open sets of real numbers $V_0, V_1, \dots, V_n, \dots$ for which the intersection $\bigcap_{n\in\mathbb{N}} V_n$ is not an open set.

3. Find closed sets of real numbers $F_0, F_1, \dots, F_n, \dots$ for which the union $\bigcup_{n\in\mathbb{N}} F_n$ is not a closed set.

(H8.3)

Prove that Theorem 2.15 Chap. 3 is not true if we drop the hypothesis that $\lim_{j\to\infty} \operatorname{diam}(A_j) = 0$ i.e., find sets or real numbers $A_1, \ldots, A_j, \ldots$ such that

$$A_0 \supseteq A_1 \supseteq \ldots \supseteq A_j \supseteq \ldots,$$

the set A_j is closed for all $j \in \mathbb{N}$ and the intersection $\bigcap_{j\in\mathbb{N}} A_j$ is the empty set.

Hint. Look for sets A_j for which $\operatorname{diam}(A_j) = \infty$ for all $j \in \mathbb{N}$.