Fachbereich Mathematik
Prof. Dr. W. Trebels
Dr. V. Gregoriades

TECHNISCHE UNIVERSITÄT DARMSTADT

2009-11-19

6th Homework Sheet Analysis I (engl.)
Winter Term 2009/10

(H6.1)

Find the radius of convergence ϱ of each of the following power series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \cdot z^n$,
2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot z^{2n}$,
3. $\displaystyle\sum_{n=0}^{\infty} \frac{n^n}{n!} \cdot z^n$.

(H6.2)

1. Let $(a_n)_{n\in\mathbb{N}}$ be a sequence in $\mathbb{C}$. Prove that the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} a_n z^n$ is the same with the radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} |a_n| z^n$.
2. Let $(a_n)_{n\in\mathbb{N}}$ be a sequence in $\mathbb{C}$ such that $a_n \to a \neq 0$. Prove that the series $\displaystyle\sum_{n=0}^{\infty} n^{10} \cdot a_n \cdot z^n$ has radius of convergence which is equal to 1.
3. Find the result of the following infinite sum

$$1 + 2 \cdot (-\frac{2}{5}) + 3 \cdot (\frac{2}{5})^2 - 4 \cdot (\frac{2}{5})^3 + \ldots + (n+1) \cdot (-1)^n \cdot (\frac{2}{5})^n + \ldots$$

Hint: You may use G6.3.

(H6.3)

Use a suitable Cauchy product of two series to find a power series $\sum_{n=0}^{\infty} a_n z^n$ with radius of convergence $\varrho > 0$ such that

$$\sum_{n=0}^{\infty} a_n z^n = \frac{1}{3 + 2z - z^2} \quad \text{for all } z \in \mathbb{C} \text{ with } |z| < \varrho.$$