

2009-10-22

2nd Home work Analysis I (engl.) Winter Term 2009/10

(H2.1)

Find all solutions $z \in \mathbb{C}$ of the equations:

(a)
$$\frac{z}{1-i} + \frac{8+i}{i-2} = \frac{1}{2}\overline{z} - 3 - 2i$$
,

(b)
$$4z + \frac{52}{z} = 24$$
 with $z \neq 0$,

(c)
$$z^2 - (3+5i)z - 16 + 4i = 0$$
.

(H2.2)

Let $A, B \subseteq \mathbb{R}$ be nonempty bounded subsets of \mathbb{R} . Prove that:

(a)
$$\sup\{a + b : a \in A, b \in B\} = \sup A + \sup B$$
,

(b)
$$\sup\{a - b : a \in A, b \in B\} = \sup A - \inf B$$
.

(H2.3)

Let $\{M_{\alpha} : \alpha \in A\}$ be a (finite or infinite) family of nonempty sets $M_{\alpha} \subset \mathbb{R}$. (It is implicit when speaking of a family of sets that the index set A is nonempty.) Let $M = \bigcup_{\alpha \in A} M_{\alpha}$ be the union of the family. Assume that M has an upper bound. Let $m_{\alpha} = \sup M_{\alpha}$. Prove that $\sup M = \sup \{m_{\alpha} : \alpha \in A\}$ holds.