Fachbereich Mathematik
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1st Home work Analysis I (engl.)

Winter Term 2009/10

(H1.1) (Cauchy-Schwarz Inequality)

(a) Prove that the following statement holds for all real numbers $x_1, x_2, \ldots, x_n$, $y_1, y_2, \ldots, y_n \in \mathbb{R}$.

$$\text{If } \sum_{k=1}^{n} x_k^2 = \sum_{k=1}^{n} y_k^2 = 1, \qquad \text{then } \sum_{k=1}^{n} x_k y_k \leq 1.$$

(b) Prove that for all $a_1, a_2, \ldots, a_n$, $b_1, b_2, \ldots, b_n \in \mathbb{R}$ we have that the Cauchy-Schwarz Inequality

$$\sum_{k=1}^{n} a_k b_k \leq \sqrt{a_1^2 + a_2^2 + \ldots + a_n^2} \cdot \sqrt{b_1^2 + b_2^2 + \ldots + b_n^2}$$

holds.

(H1.2)

Prove the following by induction. For all $n \in \mathbb{N}$ we have:

(a) $5^n - 1$ is divisible by 4.

(b) $3^{2^n} - 1$ is divisible by 2^{n+2}.

(c) The number A_n of subsets of a set with n elements is given by $A_n = 2^n$.

(H1.3)

Prove that

(a) $\displaystyle\sum_{k=1}^{n} (-1)^k k^2 = (-1)^n \binom{n+1}{2}$, for all $n \in \mathbb{N}$.

(i) $\displaystyle\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$ for all $n \in \mathbb{N}$.