Fachbereich Mathematik
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15th Exercise Sheet Analysis I (engl.)

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(G15.1)

We consider the function $f : [0,1] \to \mathbb{R}$ with $f(x) = x^2$, $x \in [0,1]$. Construct a sequence of step functions which converges uniformly to f on $[0,1]$, and use this to determine $\int_0^1 f(x)\,\mathrm{d}x$.

Hint: Consider the following result from (T2.2):

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

(G15.2)

Let $a < b \in \mathbb{R}$. Prove that every monotone real function on the interval $[a,b] \subseteq \mathbb{R}$ is jump continuous.

(G15.3)

Suppose that the function $f : [-1,1] \to \mathbb{R}$ is continuous at all $x \neq 0$ and that for all continuous functions $\varphi : [-1,1] \to \mathbb{R}$ the integral $\int_{-1}^{1} f(x) \cdot \varphi(x) dx$ equals to 0. Prove that $f(x) = 0$ for all $x \neq 0$.