TECHNISCHE
UNIVERSITÄT
DARMSTADT

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13th Exercise Sheet Analysis I (engl.)
Winter Term 2009/10

(G13.1)

Determine an approximate value of $\sqrt{2} = \frac{7}{5}(1 - \frac{1}{50})^{-1/2}$ via an appropriate Taylor polynomial such that the error is smaller than or equal to 10^{-5}.

(G13.2)

1. Compute the limit
$$\lim_{x \to 0^+} x \cdot e^{1/x}.$$

2. Define $f(x) = \dfrac{x}{x^2 + 1}, \quad x \in \mathbb{R}$. Find the intervals I of $\mathbb{R}$ for which the function f restricted on I is convex.

3. Assume that we have a function $f : \mathbb{R} \to \mathbb{R}$ which has the property

$$\frac{f(y) - f(x)}{y - x} \le \frac{f(z) - f(x)}{z - x}.$$

   for all $x < y < z$. Prove that f is convex.

(G13.3)

Define $f : \mathbb{R} \setminus \{1\} \to \mathbb{R} : f(x) = \frac{1}{1-x}$. For each $a \in \mathbb{R} \setminus \{1\}$ and $n \in \mathbb{N}$ determine the nth-Taylor polynomial $T_n f$ of f near a.