Fachbereich Mathematik
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12th Exercise Sheet Analysis I (engl.)

Winter Term 2009/10

(G12.1)

Decide where the following functions are differentiable, and find the derivative where it exists.

1. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2 \mathrm{e}^{\sin x}$.
2. The function $h(x) = x^x,\ x > 0$.
3. The function $g : \mathbb{R} \to \mathbb{R}$ is defined by

$$g(x) := \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Is g' continuous in the points where g is differentiable?

(G12.2)

1. (a) Prove that $\sqrt{1+x} < 1 + \frac{x}{2},\ x > 0$.
   (b) Prove that $e^a(b-a) < e^b - e^a < e^b(b-a)$ if $a < b$.
2. Let $f : [0,1] \to \mathbb{R}$ be a differentiable function and assume that $f(1) = 0$. Prove that there is some $\xi \in (0,1)$ such that $\xi \cdot f'(\xi) + f(\xi) = 0$.

(G12.3)

Let $f : [a,b] \to \mathbb{R}$ be differentiable and such that

$$|f(x)| + |f'(x)| \neq 0$$

for all $x \in [a,b]$. Prove that f has only finitely many zero's, i.e. that there are only finitely many $x \in [a,b]$ s.t. $f(x) = 0$.