Fachbereich Mathematik
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# 11th Exercise Sheet Analysis I (engl.)
## Winter Term 2009/10

**(G11.1)**

Recall the definitions of the *hyperbolic cosine* cosh and of the *hyperbolic sine* sinh:

$$\cosh(z) = \frac{1}{2} \cdot (e^z + e^{-z}), \quad z \in \mathbb{C}, \qquad \sinh(z) = \frac{1}{2} \cdot (e^z - e^{-z}), \quad z \in \mathbb{C}.$$

Prove that

$$\cos(z + \frac{\pi}{2}) = -\sin z, \qquad z \in \mathbb{C},$$

and that

$$\cosh^2 z - \sinh^2 z = 1, \qquad z \in \mathbb{C}.$$

**(G11.2)**

1. Calculate $\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\mathrm{i}\right)^{517}$ and sketch the result in the complex plane.
2. Sketch the 5th roots of unity in the complex plane.

**(G11.3)**

Let $M$ be a bounded subset of $\mathbb{R}$ and let $f : M \to \mathbb{R}$ be a *uniformly continuous* function. Prove that there exists a unique continuous extension $F : \overline{M} \to \mathbb{R}$ of $f$ on $M$ i.e., $F(x) = f(x)$ for all $x \in M$.

(This is direction (ii) $\Longrightarrow$ (i) of Theorem 3.16 Chap. 3 of the script).