Fachbereich Mathematik
Prof. Dr. W. Trebels
Dr. V. Gregoriades

TECHNISCHE UNIVERSITÄT DARMSTADT

2009-12-17

10th Exercise Sheet Analysis I (engl.)

Winter Term 2009/10

(G10.1)

Fill in the following table and give a small proof.

	Closed	Bounded	Compact	Open
$(0,1) \subseteq \mathbb{R}$				
$[1,2] \subseteq \mathbb{R}$				
$[1,2] \cup [3,4] \subseteq \mathbb{R}$				
$\mathbb{R} \setminus \{1\} \subseteq \mathbb{R}$				
$\mathbb{N} \subseteq \mathbb{R}$				
$[1,2) \subseteq \mathbb{R}$				

(G10.2)

1. Prove that $|\sin(x)| \leq |x|$ for all $x \in \mathbb{R}$.
2. Prove that the functions $\sin(x), \cos(x), x \in \mathbb{R}$, are uniformly continuous.

(G10.3)

Assume that we are given non-empty closed sets $A_1 \supseteq A_2 \supseteq \ldots \supseteq A_n \supseteq \ldots$ and assume also that for all $x \in A_1$ we have that $|x| \leq 100$. Prove that $\bigcap_{n\in\mathbb{N}} A_n \neq \emptyset$.