Fachbereich Mathematik
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6th Exercise Sheet Analysis I (engl.)

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(G6.1)

Find the radius of convergence for each of the following power series:

1. $\displaystyle\sum_{n=1}^{\infty} n \cdot z^{n-1}$,
2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n} \cdot z^{2n}$,
3. $\displaystyle\sum_{n=0}^{\infty} \frac{n}{(n+1)^2} \cdot z^n$.

(G6.2)

Let $(a_n)_{n\in\mathbb{N}}$ be a sequence in $\mathbb{C}$ and assume that the power series $\displaystyle\sum_{k=0}^{\infty} a_{2k} z^{2k}$, $\displaystyle\sum_{k=0}^{\infty} a_{2k+1} z^{2k+1}$ are convergent with radii of convergence R_1, R_2 respectively. Prove that the power series $\displaystyle\sum_{n=0}^{\infty} a_n z^n$ has radius of convergence $\min\{R_1, R_2\}$.

(G6.3)

Use a suitable Cauchy product of two series to find a power series $\sum_{n=0}^{\infty} a_n z^n$ with radius of convergence $\varrho > 0$ such that

$$\sum_{n=0}^{\infty} a_n z^n = \frac{1}{(1+z)^2} \quad \text{for all } z \in \mathbb{C} \text{ with } |z| < \varrho.$$