Fachbereich Mathematik
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2nd Exercise sheet Analysis I (engl.)

Winter Term 2009/10

(G2.1)

(a) Calculate $(12+5i)(2+3i)$ and $\overline{z}$, $\mathrm{Re} z$, $\mathrm{Im} z$, $\mathrm{Re}\dfrac{1}{z}$ and $\mathrm{Im}\dfrac{1}{z}$ for $z := \dfrac{12+5i}{2+3i}$.

(b) Show that $\left|\frac{1+it}{1-it}\right| = 1$ for all $t \in \mathbb{R}$.

(c) Make sketches of the following sets in the Gaussian plane.

$$M_1 := \{z \in \mathbb{C} : |z-1| \leq 1\}, \qquad M_2 := \{z \in \mathbb{C} : |z-1| \leq |z+1|\}.$$

(G2.2)

Prove Lemma 1.21 from chapter 1 in the lecture notes:

Let $M \subset \mathbb{R}$ and $-M := \{-m : m \in M\}$. Then the following statements hold:

(a) M is bounded from below $\Leftrightarrow$ $-M$ is bounded from above.

(b) Every nonempty set M that is bounded from below has an infimum. The infimum is uniquely determined and denoted by $\inf M$.

(c) $M \neq \emptyset$ is bounded from below $\Rightarrow \inf M = -\sup(-M)$.

(G2.3)

Decide whether the following sets of real numbers are bounded, and determine the supremum, infimum, maximum and minimum of each in case these exist.

(a) $A := \left\{x + \frac{1}{x} : \frac{1}{2} < x \leq 2\right\}$.

(b) $B := \{x \in \mathbb{R} : \text{ There exists a } y \in \mathbb{R} \text{ with } (x+2)^2 + 4y^2 < 9\}$.