Fachbereich Mathematik
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1st Exercise sheet Analysis I (engl.)
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(G1.1)

Prove the following statement on the basis of the axioms of $\mathbb{R}$.

Let $a, b \in \mathbb{R}$ and $a \neq 0$. If x, y are real numbers with $a \cdot x = b$ and $a \cdot y = b$, then $x = y$.

(G1.2) (Inverse Triangle Inequality)

Prove: For $x, y \in \mathbb{R}$ we have

$$\Big||x| - |y|\Big| \leq \Big|x - y\Big|.$$

(G1.3)

Prove the following for $x, y \in \mathbb{R}$.

(a) If $x < y$, then $x < \frac{x+y}{2} < y$.

(b) $\dfrac{x}{y} + \dfrac{y}{x} \geq 2, \quad$ for all $x, y \in \mathbb{R}$, $x, y > 0$.

(c) Let x, y be real numbers with $x < y$. Then there exists a real number z with $x < z < y$.

(G1.4) (Induction)

(a) We consider a chessboard with each side of length 2^n (where the sides of each square are of length 1), and we remove one arbitrary square. Prove that one can exactly cover the chessboard (minus the one square) with non-overlapping “L”-shaped pieces of cardboard, each piece covering three squares.

(b) Prove that for any natural number n the number $2^{2n} - 1$ is divisible by 3.