Fachbereich Mathematik
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14th Tutorial Analysis I (engl.)

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(T14.1)

Let $M \subset \mathbb{R}$ and let $(f_n)_{n\in\mathbb{N}}$ be a sequence of real-valued, uniformly continuous functions, which converge uniformly on M to $f : M \to \mathbb{R}$. Show that f is uniformly continuous.

(T14.2)

(a) Prove Dini's Theorem:

Let $K \subset \mathbb{R}$ be a compact set; let $(f_n)_{n\in\mathbb{N}}$ be a sequence of real-valued continuous functions converging pointwise and monotone to a continuous limit function $f : K \to \mathbb{R}$ (monotonicity means in the decreasing case $f_n(x) \geq f_{n+1}(x)$ for each $x \in K$, $n \in \mathbb{N}$). Then $(f_n)_{n\in\mathbb{N}}$ converges uniformly on K to f.

(b) Show that compactness of K is essential in Dini's Theorem by considering the example $f_n : (0, 1) \to \mathbb{R}$, $f_n(x) = (1 + nx)^{-1}$.