Fachbereich Mathematik
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13th Tutorial Analysis I (engl.)
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(T13.1) Leibniz' formula

1. Let $D \subset \mathbb{R}$ be open, $f, g : D \to \mathbb{R}$ be two n-times differentiable functions. Prove Leibniz' formula
$$\frac{d^n}{dx^n}(f \cdot g)(x) = \sum_{k=0}^{n} \binom{n}{k} f^{(n-k)}(x) g^{(k)}(x)\,. \tag{1}$$
Hint. Proceed analogously to the derivation of the binomial formula.

2. Calculate for $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto x^3 e^x$ the thousandth derivative $f^{(1000)}$.

(T13.2)

Consider the function $g : \mathbb{R} \to \mathbb{R}$ with

$$g(x) = \begin{cases} e^{-1/x^2}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

(a) Show that g is infinitely often differentiable on all of $\mathbb{R}$. Further, calculate $g^{(n)}(0)$ for all $n \in \mathbb{N}$ and determine the Taylor series of g in the origin. Is it the case that $\lim_{n\to\infty}(R_n g)(x, 0) = 0$ holds in a neighborhood of 0?

(b) Determine all the zeros as well as all global and local extremal values of g. Further calculate
$$\lim_{x\to\infty} g(x) \quad \text{and} \quad \lim_{x\to-\infty} g(x)$$
and sketch the graph of g.