



## 12th Tutorial Analysis I (engl.) Winter Term 2009/10

### (T12.1)

A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is called *even* (resp. *odd*) if for all  $x \in \mathbb{R}$  we have  $f(x) = f(-x)$  (resp.  $f(x) = -f(-x)$ ). Show that:

- (a) If  $f$  is differentiable and even, then  $f'$  is odd.
- (b) If  $f$  is differentiable and odd, then  $f'$  is even.

### (T12.2)

- (a) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function which satisfies  $|f(x)| \leq x^2$  for all  $x \in \mathbb{R}$ . Prove that  $f$  is differentiable in  $x = 0$  and find  $f'(0)$ .
- (b) Give an example of a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  which is differentiable in  $x = 0$  but discontinuous in every other point.

### (T12.3)

Let  $f : [a, b] \rightarrow \mathbb{R}$  be a function with  $f(a) = f(b) = 0$  which is continuous on  $[a, b]$  and differentiable on  $(a, b)$ . Prove that there exists a number  $\xi \in (a, b)$  with  $f'(\xi) = f(\xi)$ .

Hint: Consider the function  $x \mapsto f(x)e^{-x}$ .