Fachbereich Mathematik
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11th Tutorial Analysis I (engl.)
Winter Term 2009/10

(T11.1)

1. We have $\cos 2x = 2\left(\frac{e^{ix}+e^{-ix}}{2}\right)^2 - 1 = 2(\cos x)^2 - 1\,, \quad x \in \mathbb{R}$.
   Proceeding analogously, express $\cos 3x$ in powers of $\cos x$.

2. With the help of the formulas in (1) calculate the values $\cos\frac{\pi}{8}$ and $\cos\frac{\pi}{6}$.
   Hint: There holds $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ and $\cos x > 0$ if $0 < x < \frac{\pi}{2}$ cf. the lecture.

3. Fill out the following scheme

x	0	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$						
$\cos x$						

(T11.2)

1. Decide whether the following limits exist, and calculate the limits if they do exist.
   (a) $\lim\limits_{x\to\infty} \frac{e^x \sin x\,(x-2)^5}{e^{2x}}$,
   (b) $\lim\limits_{x\to 0^+} x^\alpha \log x, \quad \alpha > 0$.

2. Determine the limit
$$\lim_{\substack{x\to 0\\ x\neq 0}} \frac{\cos x - 1}{x^2}.$$

(T11.3)

1. Let $z \neq 0$, $z \in \mathbb{C}$. How reads the reciprocal of $1/z$ in polar coordinates?

2. Write the complex numbers $z_1 = 2 + 2i$, $z_2 = 1 + \sqrt{3}i$ and $z_1 z_2$ in polar coordinates.

3. Name all solutions of the equation $\quad (z-1)^6 = -64.$