Fachbereich Mathematik
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10th Tutorial Analysis I (engl.)
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(T10.1)

We define a sequence of sets in the following way:

$$\begin{aligned} C_0 &:= [0,1], \\ C_1 &:= C_0 \setminus \left(\frac{1}{3}, \frac{2}{3}\right), \\ C_2 &:= C_1 \setminus \left(\left(\frac{1}{9}, \frac{2}{9}\right) \cup \left(\frac{7}{9}, \frac{8}{9}\right)\right), \\ \vdots & \quad \vdots \quad \vdots \end{aligned}$$

In general we construct C_{n+1} from C_n by removing the open middle third of each of the 2^n intervals of which C_n consists. The *Cantor set* C is now given by

$$C := \bigcap_{n=0}^{\infty} C_n.$$

Prove that C is compact and that we for the interior C° of C have $C^\circ = \emptyset$.

(T10.2)

Let $X, Y \subseteq \mathbb{R}$ be nonempty. We call a mapping $f : X \to Y$ a *homeomorphism* if f is bijective and if both f and f^{-1} are continuous. Prove that if $f : X \to Y$ is bijective and continuous and if X is compact then f is a homeomorphism.