Fachbereich Mathematik
Prof. Dr. W. Trebels
Dr. V. Gregoriades

TECHNISCHE UNIVERSITÄT DARMSTADT

2009-12-08

9th Tutorial Analysis I (engl.)

Winter Term 2009/10

(T9.1)

Let $f : (-1, \infty) \to \mathbb{R},\ f(x) = \dfrac{x}{1+x}$ be given. Determine domain and range in such a way that (if necessary the restriction of) the function is bijective. Present the inverse function in an explicit way. Is the inverse continuous?

(T9.2)

Remains Theorem 2.15 Chap. 3 true, if one drops the hypothesis that the A_j , $j \in \mathbb{N}$, are closed?

(T9.3)

Sketch the following "unit balls" in $\mathbb{R}^2$:

$$\{\overrightarrow{x} = (x_1, x_2) \in \mathbb{R}^2 : |x_1| + |x_2| < 1\}, \quad \{\overrightarrow{x} : \sqrt{x_1^2 + x_2^2} < 1\}, \quad \{\overrightarrow{x} : \max(|x_1|, |x_2|) < 1\}.$$

(T9.4)

Let $m \in \mathbb{N}$ and let $f : \mathbb{R}^n \to \mathbb{R}$ be a continuous function such that:

1. $f(x) = 0 \implies x = 0$,
2. $f(tx) = t^m f(x)$ for $x \in \mathbb{R}^n$, $t > 0$.

Prove that there exists a constant $C > 0$ such that

$$|f(x)| \geq C|x|^m, \quad x \in \mathbb{R}^n.$$

Hint: Consider f on the set $\{x \in \mathbb{R}^n : |x| = 1\}$ and apply Theorem 3.9 Chap. 3.