Fachbereich Mathematik
Prof. Dr. W. Trebels
Dr. V. Gregoriades

TECHNISCHE UNIVERSITÄT DARMSTADT

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8th Tutorial Analysis I (engl.)

Winter Term 2009/10

(T8.1)

Let $(x_n)_{n\in\mathbb{N}}$ be recursively defined by

$$x_1 = -\frac{1}{2} \qquad \text{and} \qquad x_{n+1} = x_n(x_n + 1).$$

Show (a) $-1 < x_n < 0$ for all $n \in \mathbb{N}$
(b) $(x_n)_n$ is increasing
(c) Does the sequence converge? If yes determine its limit.

(T8.2)

(a) Determine the supremum and the infimum of the following sets

$$M_1 := (-2, 5) \cup (-4, 1)\,, \quad M_2 := \left\{ \frac{(-1)^n}{n} : n \in \mathbb{N} \right\}, \quad M_3 := \bigcap_{n\in\mathbb{N}} (-1 - \frac{1}{n}, 1 + \frac{1}{n})\,.$$

(b) Name the limes inferior and the limes superior of $(x_n)_{n\in\mathbb{N}}$, $x_n = (-1)^n(1 + \frac{1}{n})$.

(T8.3)

(a) Determine if the following series converge.

$$\text{(i)} \quad \sum_{n=1}^{\infty} \frac{(n+1)2^n}{n!}\,, \qquad \text{(ii)} \quad \sum_{n=2}^{\infty} \frac{n^2+1}{n^3-1}$$

(b) Calculate the radii of convergence of the following power series

$$\text{(iii)} \quad \sum_{n=0}^{\infty} (4n^3 - 3n^4) z^n\,, \qquad \text{(iv)} \quad \sum_{n=0}^{\infty} \frac{z^{2n}}{(4 + (-1)^n)^{3n}}\,, \quad z \in \mathbb{C}.$$

Voluntary supplemental problem.

(T8.4)

Find the boundary, the interior, the closure and the accumulation points of the following sets.

(a) $\mathbb{Q} \cap [0, 1]$,

(b) $\displaystyle\bigcup_{n=1}^{\infty} \left(\frac{1}{n+1}, \frac{1}{n} \right)$.