Fachbereich Mathematik
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# 7th Tutorial Analysis I (engl.)

## Winter Term 2009/10

**(T7.1)**

Let $\sum_{n=0}^{\infty} a_n z^n$ be a power series with radius of convergence $r \in (0, \infty)$.

(a) Show that the power series $\sum_{n=0}^{\infty} \frac{a_n}{n!} z^n$ has radius of convergence $\infty$.

(b) Let $f(z) := \sum_{n=0}^{\infty} \frac{a_n}{n!} z^n$ for $z \in \mathbb{C}$. Prove that for each $s \in (0, r)$ there exists a constant $M(s) > 0$ s.t.

$$|f(z)| \leq M(s) \exp(|z|/s).$$

**(T7.2)**

Let $f : \mathbb{R} \to \mathbb{R}$ be a function and let $x_0 \in \mathbb{R}$. Assume that $f$ is continuous at $x_0$ and that $f(x_0) > 0$. Prove that there exists $\delta > 0$ such that $f(x) > 0$ for all $x \in (x_0 - \delta, x_0 + \delta)$.

**(T7.3)**

Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(0) = 1$ and such that $f(x + y) \leq f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$. Prove that if $f$ is continuous at $0$ then $f$ is continuous.