Fachbereich Mathematik
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6th Tutorial Analysis I (engl.)

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(T6.1)

Decide whether the following series are convergent. Decide also whether they are absolutely convergent.

1. $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$,
2. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n+1}{n(n+1)}$,
3. $\sum_{n=1}^{\infty} \frac{2n-1}{(\sqrt{2})^n}$.

(T6.2)

Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of positive real numbers. Assume that the series $\sum_{n=1}^{\infty} a_n$ converges. Does it follow that also the series

$$\sum_{n=1}^{\infty} a_n^2$$

and

$$\sum_{n=1}^{\infty} \frac{1}{a_n}$$

converge?

(T6.3)

Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of positive real numbers. Define

$$d_n := n\left(1 - \frac{a_{n+1}}{a_n}\right).$$

Prove:

1. (*Raabe's criterion*) If there exist $N_0 \in \mathbb{N}$ and $\beta > 1$ such that $d_n \geq \beta$ for all $n \geq N_0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   Hint: First show that

   $$(\beta - 1)a_n \leq (n-1)a_n - na_{n+1}, \quad n \geq N_0.$$

2. The hypothesis of the ratio test implies the hypothesis of the Raabe criterion. In other words: If one can show that a series $\sum_{n=1}^{\infty} a_n$ converges by using the ratio test, then one can also show this by using Raabe's criterion.