Fachbereich Mathematik
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5th Tutorial Analysis I (engl.)

Winter Term 2009/10

(T5.1) (Babylonian method for computing square roots)

For $a \geq 1$ we consider the function

$$f : [1, a] \to \mathbb{R}, \quad x \mapsto \frac{1}{2}\Big(x + \frac{a}{x}\Big).$$

Show that $\sqrt{a}$ is a fixed point of this function. Estimate also the value of $\sqrt{2}$ with an error no larger than $3 \cdot 10^{-3}$. (Here it is not enough to compare with a result obtained by calculator. One has to supply a proof that the estimate has the desired accuracy without relying on knowing the true value of $\sqrt{2}$).

(T5.2)

Let $(a_n)_{n\in\mathbb{N}}$ be a bounded sequence of real numbers and let $b_n := \sup\{a_j : j \geq n\}$, $n \in \mathbb{N}$. Show that the sequence $(b_n)_{n\in\mathbb{N}}$ is monotone decreasing and convergent and that we have

$$\limsup_{n\to\infty} a_n = \lim_{n\to\infty} b_n = \lim_{n\to\infty} \sup_{j\geq n} a_j.$$

Remark: Analogously one can prove that

$$\liminf_{n\to\infty} a_n = \lim_{n\to\infty} \inf_{j\geq n} a_j.$$

The above formulas serve to motivate the notations lim inf and lim sup and are also frequently used to define these notions.

(T5.3)

(a) Calculate $\displaystyle\sum_{k=11}^{23} \sum_{j=0}^{3001} \binom{3001}{j} (-1)^{j+1} k^j \cdot 9^{3001-j} - \sum_{\ell=1}^{14} \ell^{3001}$.

(b) Let $(a_n)_{n\in\mathbb{N}}$ be a null sequence in $\mathbb{C}$, and let $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{C}$ with $\lambda_1 + \lambda_2 + \lambda_3 = 0$. Show that

$$\sum_{n=0}^{\infty} (\lambda_1 a_{n+1} + \lambda_2 a_{n+2} + \lambda_3 a_{n+3}) = \lambda_1 a_1 + (\lambda_1 + \lambda_2) a_2.$$