Fachbereich Mathematik
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4th Tutorial Analysis I (engl.)

Winter Term 2009/10

(T4.1)

Let X and Y be sets, and let $f : X \to Y$ be a mapping. Let furthermore $A, B \subseteq X$.

(a) Show that $f(A \cup B) = f(A) \cup f(B)$.

(b) Show that $f(A \cap B) \subseteq f(A) \cap f(B)$.

(c) Give an example where $f(A \cap B) \neq f(A) \cap f(B)$.

(T4.2)

Let $(a_n)_{n\in\mathbb{N}_0}$ be a sequence of complex numbers. Decide for each of the following five criteria whether they imply that $(a_n)_{n\in\mathbb{N}_0}$ is a null sequence: For every $\varepsilon > 0$ there exists an $N_0 \in \mathbb{N}$ such that we for all $n \geq N_0$ have:

(a) $|a_n + a_{n+1}| < \varepsilon$,

(b) $|a_n| < 2\varepsilon^4$,

(c) $|a_n \cdot a_{n+1}| < \varepsilon$,

(d) $|a_n^2 + a_n| < \varepsilon$,

(e) $|a_n \cdot a_{n+m}| < \varepsilon$ for all $m \in \mathbb{N}$.

Give in each case a proof or a counterexample.

(T4.3)

(a) Prove the *Sandwich Theorem*:

Let $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$ and $(c_n)_{n\in\mathbb{N}_0}$ be real sequences. If $(a_n)_{n\in\mathbb{N}_0}$ and $(b_n)_{n\in\mathbb{N}_0}$ are convergent with $\lim_{n\to\infty} a_n = a = \lim_{n\to\infty} b_n$ for some $a \in \mathbb{R}$, and if there exists an $n_0 \in \mathbb{N}$ s.t.

$$a_n \leq c_n \leq b_n \text{ for all } n \geq n_0,$$

then also $(c_n)_{n\in\mathbb{N}_0}$ is convergent, and $\lim_{n\to\infty} c_n = a$.

(b) Find the following limit by using the theorem above:

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+3}}.$$