Fachbereich Mathematik
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TECHNISCHE
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3rd Tutorial Analysis I (engl.)
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(T3.1)

Find all complex numbers $z \in \mathbb{C}$ with the property that $z^3 = 1$, and draw a sketch of these in the Gaussian plane.

(T3.2)

Recall that $\mathbb{N} := \mathbb{N}_0 \setminus \{0\}$. Decide whether the following sets are bounded, and find any suprema, infima, maxima and minima.

(a) $A := \{2^m + n! : m, n \in \mathbb{N}\}$.

(b) $B := \left\{\frac{1}{n+1} + \frac{1+(-1)^n}{2n} : n \in \mathbb{N}\right\}$.

(c) $C := \left\{\frac{|x|}{|x|+1} : x \in \mathbb{R}\right\}$.

(T3.3)

Let $M_1, M_2 \subset \mathbb{R}$ be bounded sets. We define

$$M_1 \cdot M_2 := \{x_1 \cdot x_2 : x_1 \in M_1, x_2 \in M_2\}.$$

Show that: If $x_1 \geq 0$ for all $x_1 \in M_1$ and $x_2 \geq 0$ for all $x_2 \in M_2$, then

$$\inf M_1 \cdot \inf M_2 \leq \inf(M_1 \cdot M_2) \leq \inf M_1 \cdot \sup M_2 \leq \sup(M_1 \cdot M_2) \leq \sup M_1 \cdot \sup M_2.$$