Fachbereich Mathematik
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2nd Tutorial Analysis I (engl.)
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(T2.1)

Let $a_1, \ldots, a_m \in \mathbb{N}$. Prove that for $n \in \mathbb{N}$ we have that

$$\text{if} \qquad \prod_{i=1}^{m}(1+a_i) > 2^n, \qquad \text{then} \qquad \sum_{i=1}^{m} a_i > n.$$

Hint: Prove first that $(1+k) \leq 2^k$, for $k \in \mathbb{N}$.

(T2.2)

Example:

Find a formula for the following sum:

$$\sum_{k=0}^{n}(2k+1) \quad (= 1+3+5+7+\ldots+(2n+1)).$$

Solution:

Let $a_k = k^2$ for $k = 0, 1, 2, 3, \ldots$. Then we have $a_1 - a_0 = 1^2 - 0^2 = 1$, $a_2 - a_1 = 2^2 - 1^2 = 3$, $a_3 - a_2 = 3^2 - 2^2 = 5$, $a_4 - a_3 = 4^2 - 3^2 = 7$ and $a_{n+1} - a_n = (n+1)^2 - n^2 = 2n+1$.

By adding up we get $1 + 3 + 5 + 7 + \ldots + (2n+1) = a_{n+1} - a_0 = (n+1)^2$.

Now find formulas for the following sums:

$$\sum_{k=1}^{n} k(k+1) \quad (= 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n \cdot (n+1));$$

$$\sum_{k=1}^{n} k^2 \quad (= 1^2 + 2^2 + 3^2 + \ldots + n^2).$$

Hint: $a_k = k^3$.

(T2.3)

Derive from the axioms the following properties of $\mathbb{N}_0$:

(a) If $n, m \in \mathbb{N}_0$, then also:

$$n + m,\ m \cdot n \in \mathbb{N}_0.$$

(b) Let $n \in \mathbb{N}_0$. Then there exists **no** $m \in \mathbb{N}_0$ with $n < m < n + 1$.

(c) Every nonempty set $M \subset \mathbb{N}_0$ contains a smallest element.