I. What is Ergodic Theory?

The notion "ergodic" is an artificial creation, and the newcomer to "ergodic theory" will have no intuitive understanding of its content: "elementary ergodic theory" neither is part of high school- or college- mathematics (as does "algebra") nor does its name explain its subject (as does "number theory"). Therefore it might be useful first to explain the name and the subject of "ergodic theory". Let us begin with the quotation of the first sentence of P. Walters' introductory lectures (1975, p. 1):

"Generally speaking, ergodic theory is the study of transformations and flows from the point of view of recurrence properties, mixing properties, and other global, dynamical, properties connected with asymptotic behavior."

Certainly, this definition is very systematic and complete (compare the beginning of our Lectures III. and IV.). Still we will try to add a few more answers to the question: "What is Ergodic Theory ?"

Naive answer: A container is divided into two parts with one part empty and the other filled with gas. Ergodic theory predicts what happens in the long run after we remove the dividing wall.

First etymological answer: εργοδης=difficult.

Historical answer:

1880	-	Boltzmann, Maxwell	-	ergodic hypothesis
1900	-	Poincaré	-	recurrence theorem
1931	-	von Neumann	-	mean ergodic theorem
1931	-	Birkhoff	-	individual ergodic theorem
1958	-	Kolmogorov	-	entropy as an invariant
1963	-	Sinai	-	billiard flow is ergodic
1970	-	Ornstein	-	entropy classifies Bernoulli shifts
1975	-	Akcoglu	-	individual L^p-ergodic theorem

Naive answer of a physicist: Ergodic theory proves that time mean equals space mean.

I.E. Farquhar's [1964] answer: "Ergodic theory originated as an offshot of the work of Boltzmann and of Maxwell in the kinetic theory of gases. The impetus provided by the physical problem led later to the development by pure mathematicians of ergodic theory as a branch of measure theory, and, as is to be expected, the scope of this mathematical theory extends now far beyond the initial field of interest. However, the chief physical problems to which ergodic theory has relevance, namely, the justification of the methods of statistical mechanics and the relation between reversibility and irreversibility have been by no means satisfactorily solved, and the question arises of how far the mathematical theory contributes to the elucidation of these physical problems."

Physicist's answer:

Reality	Physical model	Mathematical consequences
A gas with n particles at time $t = 0$ is given.	The "state" of the gas is a point x in the "state space" $X = \mathbb{R}^{6n}$.	
Time changes	Time change is described by the Hamiltonian differential equations. Their solutions yield a mapping $\varphi : X \to X$, such that the state x_0 at time $t = 0$ becomes the state $x_1 = \varphi(x_0)$ at time $t = 1$.	**Theorem of Liouville:** φ preserves the (normalized) Lebesgue measure μ on X.
the long run behavior is observed.	**Definition:** An observable is a function $f : X \to \mathbb{R}$, where $f(x)$ can be regarded as the outcome of a measurement, when the gas is in the state $\in X$. **Problem:** Find $\lim f(\varphi^n(x))$!	
1st objection: Time change is much faster than our observations. **2nd objection:** In practice, it is impossible to determine the state x.	**Modified problem:** Find the time mean $M_t f(x) := \lim \frac{1}{n} \sum_{i=0}^{n-1} f(\varphi^i(x))$! **Additional hypothesis** (ergodic hypothesis): Each particular motion will pass through every state consistent with its energy (see P.u.T. Ehrenfest 1911).	**"Theorem" 1:** If the ergodic hypothesis is satisfied, we have $M_t f(x) = \int f \, \mathrm{d}\mu =$ space mean, which is independent of the state x. **"Theorem" 2:** The ergodic hypothesis is "never" satisfied.

Ergodic theory looks for better ergodic hypothesis and better "ergodic theorems".

Commonly accepted etymological answer:

ἔργον = energy

–ὁδός = –path (P. and T. Ehrenfest 1911, p. 30)

"Correct" etymological answer:

ἔργον = energy

–ῶδης = –like (Boltzmann 1884/85, see also III.)

K. Jacobs' [1965] answer:
"... als Einführung für solche Leser gedacht, die gern einmal erfahren möchten, womit sich diese Theorie mit dem seltsamen, aus den griechischen Wörtern εργον (Arbeit) und οδος (Weg) zusammengesetzten Namen eigentlich beschäftigt. Die Probleme der Ergodentheorie kreisen um einen Begriff, der einerseits so viele reizvolle Spezialfälle umfaßt, daß sowohl der Polyhistor als auch der stille Genießer auf ihre Kosten kommen, andererseits so einfach ist, daß sich die zentralen Ergebnisse und Probleme der Ergodentheorie leicht darstellen lassen; diese einfach zu formulierenden Fragestellungen erfordern jedoch bei naherer Untersuchung oft derartige Anstrengungen, daß harte Arbeiter hier ihr rechtes Vergnügen finden werden."

J. Dieudonne's [1977] answer:
"Le point de départ de la théorie ergodique provient du développement de la mécanique statistique et de la theorie cinétique des gaz, où l'expérience suggère und tendence à l'"uniformite": si l'on considère à un instant donné un mélange hétérogène de plusieurs gaz, l'évolution du mélange au cours du temps tend à le rendre homogéne."

W. Parry's [1981] answer:
"Ergodic Theory is difficult to characterize, as it stands at the junction of so many areas, drawing on the techniques and examples of probability theory, vector fields on manifolds, group actions on homogeneous spaces, number theory, statistical mechanics, etc..."" (e.g. functional analysis; added by the authors).

Elementary mathematical answer:
Let X be a set, $\varphi : X \to X$ a mapping. The induced operator T_φ maps functions $f : X \to \mathbb{R}$ into $T_\varphi f := f \circ \varphi$. Ergodic theory investigates the asymptotic behavior of φ^n and T_φ^n for $n \in \mathbb{N}$.

Our answer:
More structure is needed on the set X, usually at least a topological or a measure theoretical structure. In both cases we can study the asymptotic behavior of the powers T^n of the linear operator $T = T_\varphi$, defined either on the Banach space $C(X)$ of all continuous functions on X or on the Banach space $L^1(X, \Sigma, \mu)$ of all μ-integrable functions on X.