Formelsammlung zur Klausur

- *Leibniz-Formel*:

$$\det(A) = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma)\, a_{\sigma(1),1} \cdot a_{\sigma(2),2} \cdot \ldots \cdot a_{\sigma(n),n}.$$

- *Cramer's Regel*:

$$x_i = \frac{\det(a_1, \ldots, a_{i-1}, b, a_{i+1}, \ldots, a_n)}{\det(A)}.$$

- *Cosinus-Satz*:

$$\cos(\gamma) = \frac{< x, y >}{\|x\|\, \|y\|}.$$

- *Taylor-Formel*:

$$f(x + \nu) = f(x) + \partial_\nu f(x) + \frac{\partial_\nu^2 f(x)}{2!} + \ldots + \frac{\partial_\nu^k f(x)}{k!} + R_{k+1}(x, \nu)$$

mit $R_{k+1}(x, \nu) = \dfrac{\partial_\nu^{k+1} f(x + \xi\nu)}{(k+1)!}$ für $\xi \in [0, 1]$.

- *Polarkoordinaten*: Transformationsfunktion:

$$T(r, \varphi) = \begin{pmatrix} r\cos(\varphi) \\ r\sin(\varphi) \end{pmatrix}.$$

Jacobi-Matrix:

$$J_T(r, \varphi) = \begin{pmatrix} \cos(\varphi) & -r\sin(\varphi) \\ \sin(\varphi) & r\cos(\varphi) \end{pmatrix},$$

$$\det(J_T(r, \varphi)) = r.$$

- *Zylinderkoordinaten*: Transformationsfunktion:

$$T(r, \varphi, t) = \begin{pmatrix} r\cos(\varphi) \\ r\sin(\varphi) \\ t \end{pmatrix}.$$

Jacobi-Matrix:

$$J_T(r, \varphi, t) = \begin{pmatrix} \cos(\varphi) & -r\sin(\varphi) & 0 \\ \sin(\varphi) & r\cos(\varphi) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\det(J_T(r, \varphi, t)) = r.$$

- *Kugelkoordinaten*: Transformationsfunktion:

$$T(r, \varphi, \gamma) = \begin{pmatrix} r\cos(\varphi)\sin(\gamma) \\ r\sin(\varphi)\sin(\gamma) \\ r\cos(\gamma) \end{pmatrix}.$$

Jacobi-Matrix:

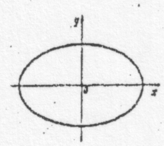
$$J_T(r, \varphi, \gamma) = \begin{pmatrix} \cos(\varphi)\sin(\gamma) & -r\sin(\varphi)\sin(\gamma) & r\cos(\varphi)\cos(\gamma) \\ \sin(\varphi)\sin(\gamma) & r\cos(\varphi)\sin(\gamma) & r\sin(\varphi)\cos(\gamma) \\ \cos(\gamma) & 0 & -r\sin(\gamma) \end{pmatrix},$$

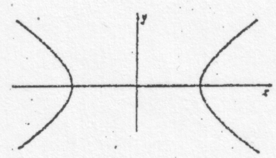
$$\det(J_T(r, \varphi, t)) = -r^2\sin(\gamma).$$

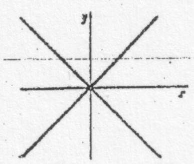
Die Normalformen der Quadriken im $\mathbb{R}_2$ (Konstante a, b, p alle $\neq 0$)

Rang $A = 2$ (Alle Eigenwerte $\neq 0$)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$	Ellipse (evtl. Kreis)
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + 1 = 0$	leere Menge
$\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$	Hyperbel
$x^2 + a^2y^2 = 0$	Punkt
$x^2 - a^2y^2 = 0$	Geradenpaar mit Schnittpunkt

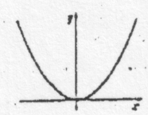


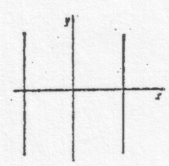


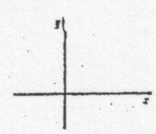


Rang $A = 1$ (Ein Eigenwert $= 0$)

$x^2 - 2py = 0$	Parabel
$x^2 - a^2 = 0$	paralleles Geradenpaar
$x^2 + a^2 = 0$	leere Menge
$x^2 = 0$	Gerade $x = 0$



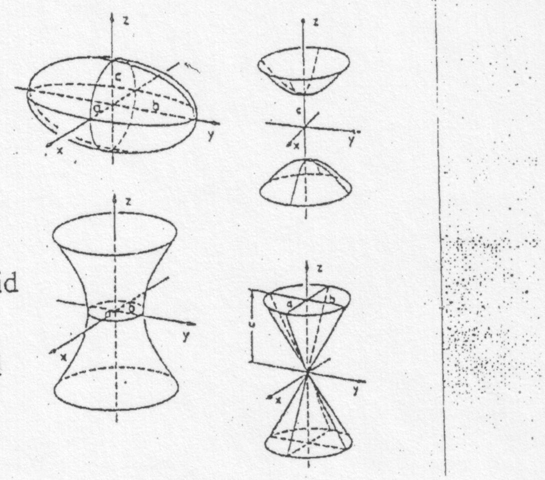




Die Normalformen der Quadriken im $\mathbb{R}_3$ (Konstante a, b, c, p alle $\neq 0$)

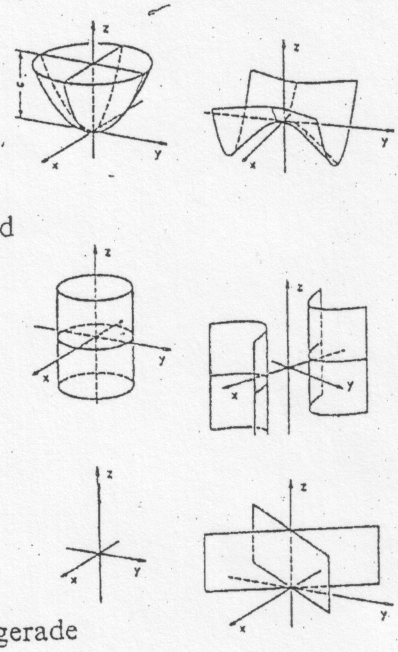
Rang $A = 3$ (Alle Eigenwerte $\neq 0$)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$	Ellipsoid (evtl. Kugel)
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} + 1 = 0$	leere Menge
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} + 1 = 0$	zweischaliges Hyperboloid
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} - 1 = 0$	einschaliges Hyperboloid
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$	Punkt
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$	Kegel



Rang $A = 2$ (Ein Eigenwert $= 0$)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2pz = 0$	elliptisches Paraboloid
$\frac{x^2}{a^2} - \frac{y^2}{b^2} - 2pz = 0$	hyperbolisches Paraboloid
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + 1 = 0$	leere Menge
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$	elliptischer Zylinder
$\frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$	hyperbolischer Zylinder
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$	Gerade
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$	Ebenenpaar mit Schnittgerade



Rang $A = 1$ (Zwei Eigenwerte $= 0$)

$x^2 - 2py = 0$	parabolischer Zylinder
$x^2 - a^2 = 0$	paralleles Ebenenpaar
$x^2 + a^2 = 0$	leere Menge
$x^2 = 0$	Ebene