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AG 06, FB Mathematik
Tech. Univ. Darmstadt

Partial Differential Equations I: Linear Theory

Tutorial 11: Exercises[1]

In this tutorial we are going to consider the double layer potential and the jump relations for some special domains.

1. Let $\Omega \subset \mathbb{R}^3$ be a bounded open set with $\partial\Omega \in C_2$. Assume $\Gamma \subset \partial\Omega$ is a part of non-zero measure, which is flat. Show that for $x_0,\ y$, we have

$$\frac{\partial}{\partial n_y} \frac{e^{i\sqrt{\lambda}|x_0-y|}}{|x_0-y|} = 0.$$

2. Let $f \in C(\partial\Omega)$ with $f(x) = 0$ for all $x \in \partial\Omega \setminus \Gamma$. Use the previous result to construct a solution $u \in C_2(\Omega) \cap C(\bar{\Omega})$ of

$$\begin{aligned} \Delta u(x) + \lambda u(x) &= 0,\ x \in \Omega, \\ u(x) &= f(x),\ x \in \Gamma. \end{aligned}$$

3. Is the function u, which you constructed, a solution of the Dirichlet problem

$$\begin{aligned} \Delta u(x) + \lambda u(x) &= 0,\ x \in \Omega, \\ u(x) &= f(x),\ x \in \partial\Omega? \end{aligned}$$

4. Let $\Omega = B_R(0) \subset \mathbb{R}^3$, let $f = \text{const}$. Show that the boundary value problem

$$\begin{aligned} \Delta u(x) &= 0,\ x \in B_R(0), \\ u(x) &= f(x),\ x \in \partial B_R(0) \end{aligned}$$

[1] If you have any opinion and/or suggestion on the Tutorial, please send your email to Prof. Dr. H.-D. Alber at alber@mathematik.tu-darmstadt.de, or to Dr. P. Zhu at zhu@mathematik.tu-darmstadt.de.

can be solved if

$$\frac{1}{2\pi}\int_{|y|=R}\frac{y\cdot(y-x_0)}{R|x_0-y|^3}dS_y \neq -1$$

for a given $x_0 \in \partial B_R(0)$.

The following problem is your homework.

5. Compute the integral which is derived from that in problem 4:

$$\frac{1}{2\pi}\int_{|z|=1}\frac{z\cdot(z-z_0)}{|z_0-z|^3}dS_z,$$

where $z_0 = x_0/R$. For simplicity, you can calculate it supposing $z_0 = (0,0,1)$. Is this assumption $z_0 = (0,0,1)$ a restriction?