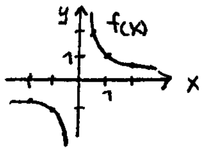
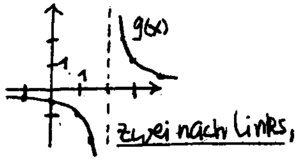
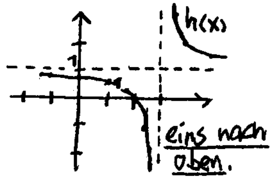
LÖSUNGEN AUFGABENBLATT NR. 5, Mathematik für „Joint Bachelor"

49.



50.





51. Nenner: $x^2-2x-3=(x+1)(x-3) \rightarrow D_f = \underline{\underline{\mathbb{R}\setminus\{-1,3\}}}$.

52. Zähler: $x^2-3x=x(x-3)$. Also stetige Fortsetzung für $x=3$: $\underline{\underline{\tilde{f}(x)=\frac{x}{x+1}}}$.

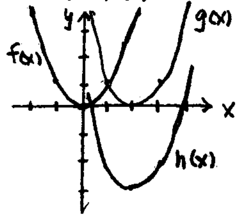
53. $\frac{f(1+1)-f(1)}{1} = \frac{2^2-1^2}{1} = \frac{4-1}{1} = \underline{\underline{3}}$.

54. $\frac{f(1+0{,}5)-f(1)}{0{,}5} = \underline{\underline{2{,}5}}$, $\frac{f(1+0{,}1)-f(1)}{0{,}1} = \underline{\underline{2{,}1}}$, $\frac{f(1-0{,}1)-f(1)}{-0{,}1} = \underline{\underline{1{,}9}}$.

55. $T(x) = f(x_0) + f'(x_0)(x-x_0) = 1^2 + 2\cdot 1\,(x-1) = \underline{\underline{2x-1}}$.

56. $T(x)=f(x) \rightarrow 2x-1=x^2 \rightarrow \underline{\underline{x=1}}$ (und $\underline{y}=x^2=\underline{1}$).

57.



58. $g(x) = x^2-4x+2 = (x-2)^2-(-2)^2+2$

$= (x-2)^2 - 2$,

also: $f(x)=x^2$ 2 nach rechts, 2 nach unten.

Liefert $g(x)=x^2-4x+2$.

59. $\underline{\Delta x = 1}$: $\frac{\Delta f}{\Delta x} = \underline{\underline{-\frac{1}{2}}}$, $\underline{\Delta x = 0{,}5}$: $\frac{\Delta f}{\Delta x} = \frac{-\frac{1}{3}}{\frac{1}{2}} = \underline{\underline{-\frac{2}{3}}}$,

$\underline{\Delta x = 0{,}1}$: $\frac{\Delta f}{\Delta x} = \frac{-\frac{1}{11}}{\frac{1}{10}} = \underline{\underline{-\frac{10}{11}}}$, $\Delta x = -0{,}1$: $\frac{\Delta f}{\Delta x} = \frac{\frac{1}{9}}{-\frac{1}{10}} = \underline{\underline{-\frac{10}{9}}}$.

$f'(x) = -\frac{1}{x^2} \rightarrow f'(1) = -1$ (also oben beste Näherung für $\Delta x = 0{,}1$).

60. a) $q(x) = p(x):(x-c)$, Polynomdivision geht auf ohne Rest, da c Nullstelle von $p(x)$.

b) Fortlaufende Anwendung des Ergebnissen aus Teil a):

$$p(x) = (x-c_1)\,q_1(x) = (x-c_1)(x-c_2)\,q_2(x) = \ldots = \underline{(x-c_1)(x-c_2)\cdot\ldots\cdot(x-c_n)\,\overbrace{q_n}^{\text{Konstante}}}.$$