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Fachbereich Mathematik
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Probability Theory
8. Aufgabenblatt

Gruppenübungen

Aufgabe G23:
Let X be a random variable. Then the following are equivalent:

(i) X is integrable.

(ii) $\sum_{n=1}^{\infty} P[|X| > n\epsilon] < +\infty$ for all $\epsilon > 0$.

Aufgabe G24:
Let X_1 and X_2 be random variables on $(\Omega, \mathcal{A}, P)$. Consider the following:

(i) (X_1, X_2) is uniformly distributed on $[0,1] \times [0,1]$,

(ii) (X_1, X_2) is uniformly distributed on the unit circle $\{(x_1, x_2) \,|\, x_1^2 + x_2^2 \leq 1\}$.

Are X_1 and X_2 independent? Determine the distributions of X_1 and X_2.

Aufgabe G25:
For the random walk S_n, $n \in \mathbb{N}$, in exercise H21 show that

$$\limsup_{n\uparrow\infty} \frac{S_n}{\sqrt{n}} = +\infty, \quad \liminf_{n\uparrow\infty} \frac{S_n}{\sqrt{n}} = -\infty \quad \text{P-a.s.}$$

Hausübungen

Aufgabe H22:

Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables. Use G23 to prove that

$$\lim_{n\to\infty} \frac{X_n}{n} = 0 \text{ P-a.s.} \quad \Longleftrightarrow \quad E[|X_1|] < +\infty.$$

Which implication remains true if we drop the assumption of independence of $(X_n)_{n\in\mathbb{N}}$.

Aufgabe H23:

Let X, Y be independent, $N(0, \sigma^2)$-distributed random variables and

$$R := \sqrt{X^2 + Y^2} \quad \text{and} \quad \Phi := \arctan \frac{Y}{X}.$$

Show that:

(i) R and Φ are independent.

(iii) The distribution of R is absolute continuous with density

$$\frac{r}{\sigma^2} \cdot e^{-\frac{r^2}{2\sigma^2}} \cdot 1_{[0,\infty[}(r).$$

Aufgabe H24:

The density of the gamma-distribution $\Gamma_{\alpha,p}$ $(\alpha > 0, p > 0)$ is given by

$$f_{\alpha,p}(x) = \begin{cases} \frac{1}{\Gamma(p)} \alpha^p x^{p-1} e^{-\alpha x} & , x > 0 \\ 0 & , x \leq 0. \end{cases}$$

Calculate the distribution of the sum of two independent random variables, which are Γ_{α,p_1}- and Γ_{α,p_2}-distributed.