

2. (12):

a) $|z_1| = \sqrt{(\sqrt{3})^2 - 2^2} = 2\sqrt{3} \rightarrow z_1 = 2\sqrt{3} \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2}\right)$

Wir betrachten $\varphi \in [0, 2\pi[$ mit

$$\cos \varphi = -\frac{1}{2} \quad \text{und} \quad \sin \varphi = \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{4}}$$

$$\Leftrightarrow (\varphi = \frac{2}{3}\pi \text{ oder } \varphi = \frac{4}{3}\pi) \quad \text{und} \quad (\varphi = \frac{1}{3}\pi \text{ oder } \varphi = \frac{2}{3}\pi)$$

$$\Leftrightarrow \varphi = \frac{2}{3}\pi$$

Also gilt: $z_1 = 2\sqrt{3} \left(\cos\left(\frac{2}{3}\pi\right) + i \sin\left(\frac{2}{3}\pi\right)\right) = 2\sqrt{3} \cdot e^{i\frac{2}{3}\pi}$

$|z_2| = \sqrt{3} \rightarrow z_2 = \sqrt{3} \left(-\frac{\sqrt{3}}{2} + i \frac{1}{2}\right)$

Suchen $\varphi \in [0, 2\pi[$ mit

$$\cos \varphi = -\frac{\sqrt{3}}{2} \quad \text{und} \quad \sin \varphi = \frac{1}{2}$$

$$\Leftrightarrow (\varphi = \frac{5}{6}\pi \text{ o. } \varphi = \frac{7}{6}\pi) \quad \text{und} \quad (\varphi = \frac{1}{6}\pi \text{ o. } \varphi = \frac{5}{6}\pi)$$

$$\Leftrightarrow \varphi = \frac{5}{6}\pi$$

Also gilt: $z_2 = \sqrt{3} \left(\cos\left(\frac{5}{6}\pi\right) + i \sin\left(\frac{5}{6}\pi\right)\right) = \sqrt{3} e^{i\frac{5}{6}\pi}$

b) $z_3 = 6 e^{i\frac{3}{2}\pi}$; $z_4 = 2 \cdot e^{i\frac{1}{6}\pi}$; $z_5 = 729 \cdot e^{i\pi} = 729$

c) $z_3 = 6 \cdot \left(\cos\left(\frac{3}{2}\pi\right) + i \sin\left(\frac{3}{2}\pi\right)\right) = -6i$

$$z_4 = 2 \left(\cos\left(\frac{1}{6}\pi\right) + i \sin\left(\frac{1}{6}\pi\right)\right) = 2 \cdot \sqrt{\frac{3}{4}} + i 2 \left(-\frac{1}{2}\right) = \sqrt{3} - i$$

$$z_5 = 729$$