Kurslösung zum 1. Übungsblatt

Zu G1:

(a) (i) $\mathrm{Re}\, z = 4$; $\mathrm{Im}\, z = 13$; $|z| = \sqrt{185}$

(ii) $z = \frac{2+2i}{1-i} - \frac{5+i}{3+i} = \frac{(2+2i)(1+i)}{(1-i)(1+i)} - \frac{(5+i)(3-i)}{(3+i)(3-i)}$

$= \frac{1+5i}{2} - \frac{16-2i}{10} = -1{,}1 + 2{,}7i$

$\Rightarrow \mathrm{Re}\, z = -1{,}1$; $\mathrm{Im}\, z = 2{,}7$; $|z| = \sqrt{9{,}5}$

(iii) $\mathrm{Re}\, z = 0$; $\mathrm{Im}\, z = -1$; $|z| = 1$

(iv) $\mathrm{Re}\, z = e^a \cos b$; $\mathrm{Im}\, z = e^a \sin b$; $|z| = e^a$

(v) Für $z = a + ib$ gilt $e^z = e^a \cos b + i\, e^a \sin b = i$

$\Rightarrow e^a \cos b = 0$ und $e^a \sin b = 1$

$\Rightarrow b = (2k+1)\frac{\pi}{2}$, $k \in \mathbb{Z}$

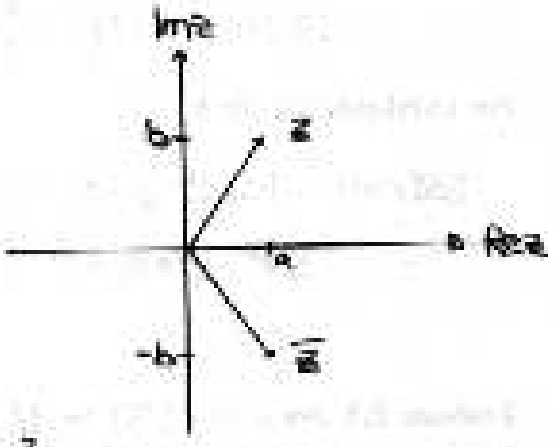
$\Rightarrow 1 = e^a \sin\left((2k+1)\frac{\pi}{2}\right) = e^a$ $\Rightarrow a = 0$

$\Rightarrow z = (2k+1)\frac{\pi}{2}\, i$, $k \in \mathbb{Z}$

Also: $\mathrm{Re}\, z = 0$, $\mathrm{Im}\, z = (2k+1)\frac{\pi}{2}$; $|z| = (2k+1)\frac{\pi}{2}$

(b)

(1) $z = a + ib \Rightarrow \bar{z} = a - ib$



(2) $z = a + ib \Rightarrow iz = ia - b$