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9. Aufgabenblatt zur Vorlesung „Probability Theory“

1. Let X_1 and X_2 be independent with $P_{X_i} = 1/2 \cdot (\varepsilon_1 + \varepsilon_{-1})$ for $i = 1, 2$. Verify that $(X_1, X_2, X_1 \cdot X_2)$ is pairwise independent but not independent.

2. Consider random variables $X_n,\ X,\ Y_n,\ Y$ on a common probability space, and assume independence of (X_n, Y_n) for every n as well as independence of (X, Y). Prove

$$X_n \xrightarrow{\mathrm{d}} X \quad \wedge \quad Y_n \xrightarrow{\mathrm{d}} Y \quad \Rightarrow \quad X_n + Y_n \xrightarrow{\mathrm{d}} X + Y.$$

Cf. Aufgabe 8.3.a).

3. **a)** Show that X and Y are independent iff

$$E(f \circ X - g \circ Y)^2 \geq E(f \circ X - E(f \circ X))^2$$

for all Borel-measurable mappings $f, g : \mathbb{R} \to \mathbb{R}$ with $f \circ X, g \circ Y \in \mathfrak{L}^2$. Interpretation in terms of prediction problems?

b) Construct a probability space together with square-integrable random variables X and Y on this space such that

- X and Y are uncorrelated but not independent, and
- $E(X - g \circ Y)^2 > E(X - E(X))^2$ for every measurable mapping g that is different from the constant mapping $E(X)$.

4. Betrachten Sie das Cramér-Lundberg-Modell mit Zeithorizont $T > 0$. Entwickeln und implementieren Sie einen Algorithmus zur Simulation und graphischen Darstellung von Pfaden des Risikoreserveprozesses $(R_t)_{t \in [0,T]}$.