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7. Aufgabenblatt zur Vorlesung „Probability Theory“

1. Consider a queue where, per time step,

- in case of a non-empty queue, the customer at the head of the queue is served and leaves,
- n new customers arrive with probability b_n for $n \in \mathbb{N}_0$.

a) Choose an appropriate measurable space to model the lengths of the queue at all times $i \in \mathbb{N}_0$. Define the corresponding transition kernel by means of an infinite-dimensional stochastic matrix $(\overline{K}(k,\ell))_{k,\ell\in\mathbb{N}_0}$, i.e., $\overline{K}(k,\ell) \geq 0$ for $k,\ell \in \mathbb{N}_0$ and $\sum_{\ell=0}^{\infty} \overline{K}(k,\ell) = 1$ for $k \in \mathbb{N}_0$.

b) Suppose that initially the queue is empty. Derive a recursive formula for the probability of length $k \in \mathbb{N}_0$ of the queue at time $i \in \mathbb{N}_0$. Derive a formula for the probability of lengths $(k_1, \dots, k_i)$ of the queue at times $1, \dots, i$.

2. Sei $(\Omega, \mathcal{A})$ ein messbarer Raum, μ ein σ-endliches und ν ein endliches Maß auf $\mathcal{A}$. Zeigen Sie:

$$\nu \ll \mu \quad \Leftrightarrow \quad \forall\, \varepsilon > 0\ \exists\, \delta > 0\ \forall\, A \in \mathcal{A}\ \Big(\mu(A) \leq \delta \Rightarrow \nu(A) \leq \varepsilon\Big).$$

Gilt diese Aussage auch dann, wenn ν nur σ-endlich ist?

3. Let $\Omega_i = \{0,1\}$, $\mathfrak{A}_i = \mathfrak{P}(\Omega_i)$, and $\mu_i = p \cdot \varepsilon_1 + (1-p) \cdot \varepsilon_0$ for $p \in]0,1[$ and $i \in \mathbb{N}$. Consider the corresponding product space $(\Omega, \mathfrak{A}, P)$.

a) Determine the distribution of the random variable

$$X_n : \Omega \to \{0, \dots, n\} : \omega \mapsto |\{i \in \{1, \dots, n\} : \omega_i = 1\}|.$$

b) Construct a random variable on $(\Omega, \mathfrak{A}, P)$ that is geometrically distributed with parameter p.

c) Construct random variables X and Y on $(\Omega, \mathfrak{A}, P)$ that do not coincide almost surely but have the same distribution.

4. Sei $(\Omega, \mathfrak{A}, P)$ ein Wahrscheinlichkeitsraum mit einer abzählbaren Menge Ω. Zeigen Sie, daß in diesem Fall die fast sichere Konvergenz äquivalent zur stochastischen Konvergenz ist.