Exercises Unit 8

1. Determine the tangent at x_0 :

    (a) $f(x) = 2x^3 - 7, \quad x_0 = -1$.

    (b) $f(x) = \dfrac{1}{x}, \quad x_0 = \dfrac{1}{2}$.

2. Does $\displaystyle\lim_{x \to x_0} \dfrac{f(x_0) - f(x)}{x_0 - x}$ exist for the following function?

$$f(x) = |x^3|, \quad x_0 = 0$$

    Use the definition of differentiability to decide if the function is differentiable in $x_0 = 0$.

3. Write the following function as a composition of simpler functions and calculate their derivatives using the chain rule: $f(x) = \sqrt{(2x^2 + x)^3 + 1}$

4. Show, that $(f \pm g)' = f' \pm g'$.

5. Use the product rule and the chain rule to prove the quotient rule.

6. Decompose a fixed real number c into two summands such that their product is maximal.