# Mathematics with Computer Science

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Introductory Course
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## Exercises Unit 7

1. Consider the function $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto x$. Show that for all $c \in \mathbb{R}$ the limit $\lim\limits_{x \to c} f(x)$ exists and that $f$ is continuous in $\mathbb{R}$.

2. For what values of the real number $c$ is the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} cx + 1 & \text{if } x \leq 3 \\ cx^2 - 1 & \text{if } x > 3 \end{cases}$$

   continuous?

3. Use the Intermediate Value Theorem to show that there exists a positive number $c$ such that $c^2 = 2$. (This proves the existence of the number $\sqrt{2}$.)

4. Show that the following equations have a root in the given interval:

   (a) $x^3 - 3x + 1 = 0$ in $[0, 1]$

   (b) $x^5 - 2x^4 - x - 3 = 0$ in $[2, 3]$

   (c) $x^2 = \sqrt{x + 1}$ in $[1, 2]$

5. Show with the $\varepsilon$-$\delta$-condition, that the following function is continous in $x_0 = 0$.

$$\begin{array}{rcl} f : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x^2 \end{array}$$

   *Hint:* $|a + b| \leq 2|a| + 1$ *if* $|a - b| \leq 1$.

6. Show, that for continous functions $f$ and $g$ the functions

   - $f \pm g$
   - $f \cdot g$
   - $\frac{f}{g}$
   - $f \circ g$

   are continous (if defined).