# Mathematics with Computer Science

Introductory Course
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## Exercises Unit 6

1. Prove, if the following series are convergent:

$$(i) \sum_{n=1}^{\infty} \frac{n+1}{n^3} \qquad (ii) \sum_{n=0}^{\infty} \frac{n!}{3^n} \qquad (iii) \sum_{k=1}^{\infty} \frac{n!}{n^n}$$

   *Hint: You can use, that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.*

2. a) Show, that for the geometric sum the following is true:

$$\sum_{k=0}^{n} x^n = \frac{1 - x^{n+1}}{1 - x}$$

   *Hint: Multiply a usefull term and use the telescope sum trick.*

   b) Determine the value of the geometric series for $x = \frac{2}{3}$.

3. Suppose for a real series $\sum_{n=0}^{\infty} a_n$ there exists a sequence $(x_n)$ with $a_n \geq x_n \geq 0$ such that $\sum_{n=0}^{\infty} x_n$ is divergent. Prove that $\sum_{n=0}^{\infty} a_n$ diverges as well.

4. Prove, if the following series are convergent:

$$(i) \sum_{n=0}^{\infty} \frac{2^n}{3^n + 8} \qquad (ii) \sum_{n=2}^{\infty} \frac{1}{n-1} \qquad (iii) \sum_{n=0}^{\infty} \frac{1}{n^2 - n + 1}$$

5. Show, that for the alternating harmonic series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

   the commutative law does not hold.