Mathematics with Computer Science

Introductory Course
Winter Semester 2008/2009

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Exercises Unit 5

1. Consider the sequence
$$a_n = \frac{2n-3}{5n+7}, \quad n \in \mathbb{N}.$$
   (a) Show that the limit of this sequence is $\frac{2}{5}$.
   (b) Which terms of the sequence are closer to $\frac{2}{5}$ than $\varepsilon = \frac{1}{10}$?

2. (a) What is the limit of the sequence $a_n = \frac{1}{2^n}$ for $n \in \mathbb{N}$ if n goes to ∞?
   (b) What is the limit of the sequence
$$\frac{1}{2}, \quad \frac{1}{2}+\frac{1}{4}, \quad \frac{1}{2}+\frac{1}{4}+\frac{1}{8}, \quad \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}, \ldots$$
   Can you give a geometric interpretation of this limit process?

3. The first terms of an infinite sequence are $1, 3, 7, 15, 31, 63$.
   (a) Find a recursive definition for the sequence.
   (b) Find an explicit definition.

4. Find a recursive definition for the sequence
$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots$$
   What is the limit of this sequence?

5. Determine the limit (if it exists) of
$$a_n = \frac{5}{n} + \frac{7n}{n^2+1}, \qquad b_n = (6+\frac{1}{n})(\frac{n+2}{2n+1}-1), \qquad c_n = \frac{2n^2-2}{3n^2-3},$$
$$d_n = \frac{\frac{1}{n^2}+\frac{1}{n^3}}{\frac{1}{n}+\frac{1}{n^2}}, \qquad e_n = \frac{2n+(-1)^n n}{n+1}.$$

6. Determine the limit (if it exists) of
   (a) $a_n = \sqrt{n^2+1} - n, \quad n \in \mathbb{N}.$
   (b) $b_n = n(\sqrt{n^2+1} - n), \quad n \in \mathbb{N}.$
   (c) $c_n = n^2(\sqrt{n^2+1} - n), \quad n \in \mathbb{N}.$