Mathematics with Computer Science

Exercises Unit 4

1. Which of the following functions are injective, surjective, bijective?

$$\begin{aligned} f_1 : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^2 \end{aligned}$$

$$\begin{aligned} f_2 : \mathbb{R}_{\geq 0} &\to \mathbb{R} \\ x &\mapsto x^2 \end{aligned}$$

$$\begin{aligned} f_3 : \mathbb{R}_{\geq 0} &\to \mathbb{R}_{\geq 0} \\ x &\mapsto x^2 \end{aligned}$$

$$\begin{aligned} f_4 : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^3 \end{aligned}$$

$$\begin{aligned} f_5 : \mathbb{R} \setminus \{0\} &\to \mathbb{R} \\ x &\mapsto \frac{1}{x} \end{aligned}$$

$$\begin{aligned} f_6 : \mathbb{R} \setminus \{0\} &\to \mathbb{R}_{>0} \\ x &\mapsto \frac{1}{x^2} \end{aligned}$$

2. Find a function $f : \mathbb{N} \longrightarrow \mathbb{N}$ which is

    (a) injective but not surjective,

    (b) surjective but not injective.

3. Find functions f and g such that the following functions can be written as $f \circ g$.

$$F_1(x) = \sqrt{x+9} \qquad\qquad F_3(x) = \sqrt{x} + 2$$
$$F_2(x) = (x-5)^2 \qquad\qquad F_4(x) = \frac{1}{x-1}$$

4. (a) Consider the functions f and g from $\mathbb{R}$ to $\mathbb{R}$ given by $f(x) = x^2$ and $g(x) = x - 3$. Find the composite functions $f \circ f$, $f \circ g$, $g \circ f$ and $g \circ g$ and determine the domain of each function. Demonstrate that $f \circ g$ is not necessarily the same as $g \circ f$.

    (b) Find $f \circ g \circ h$ where $f(x) = x/(x+1)$, $g(x) = x^2$ and $h(x) = x + 3$. Find the maximal subset of $\mathbb{R}$ on which $f \circ g \circ h$ is defined.

5. Find all zeros of the following functions:

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto x^3 - 6x^2 + 11x - 6 \end{aligned}$$

$$\begin{aligned} g : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto x^4 - 4x^3 + 6x^2 - 4x + 1 \end{aligned}$$

$$\begin{aligned} h : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto x^4 - 1 \end{aligned}$$

6. Determine the set

$$\{x \in \mathbb{R} \ : \ f(x) = 1\},$$

    with

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto x^3 - x^2 - 4x + 5 \end{aligned}$$