Mathematics with Computer Science

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Introductory Course
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Exercises Unit 2

1. (a) Verify that the inversion formula

$$(a+bi)^{-1} = \frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}i$$

is correct.

(b) Compute $\frac{5-3i}{3+2i}$.

(c) Try to find the solution of $(2-i)\cdot(2-2i)$ geometrically.

(d) Let $c = a+bi$ be a complex number. Compute $(x-c)(x-\bar{c})$. Can you guess a solution for x^2-2x+2?

2. Let A and B be propositions. Show that the following statments are true by setting up truth tables in each case:

(a) A is the same as $\neg(\neg A)$.

(b) $\neg(A \wedge B)$ is the same as $\neg A \vee \neg B$.

(c) $\neg(A \vee B)$ is the same as $\neg A \wedge \neg B$.

(d) $A \Rightarrow B$ is the same as $\neg A \vee B$.

3. Show: For propositions A and B we have that $A \wedge (A \Rightarrow B)$ implies B. Interpret this rule.

4. Find examples of implications that are not equivalences and explain why the conclusion works in one direction only.

5. Prove the proposition

$$(A \implies B) \iff (\neg B \implies \neg A)$$

6. Negate the following propositions:

(a) All mathematicians are smokers.

(b) No students like to go to parties.

(c) All bananas are yellow.

(d) There exists a black swan.

7. Negate the following propositions:

(a) $\exists x \in S : A(x) \wedge B(x)$

(b) $\forall x \in S : A(x) \Rightarrow B(x)$

8. Write the following propositions as a formal expression using quantors, negate the formal expression and convert the negation back into everyday language.

(a) For every real number x there exists a natural number n such that $n > x$.

(b) There is no rational number x satisfying the equation $x^2 = 0$.