Mathematics with Computer Science

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Fachbereich Mathematik
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Introductory Course
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Hints to Exercises Unit 9

1. The area is 24. A sketch of the graphs shows what one has to do.
2. With $f(x) := -\cos x$ and $g(x) := \sin x$ we obtain on the one hand
$$\int_{-\pi/2}^{\pi/2} \sin^2 x\, dx = -\cos x \sin x\Big|_{-\pi/2}^{\pi/2} + \int_{-\pi/2}^{\pi/2} \cos^2 x\, dx = \int_{-\pi/2}^{\pi/2} \cos^2 x\, dx;$$
on the other hand, $\sin^2 x + \cos^2 x = 1$, and so
$$\int_{-\pi/2}^{\pi/2} \sin^2 x\, dx + \int_{-\pi/2}^{\pi/2} \cos^2 x\, dx = \int_{-\pi/2}^{\pi/2} 1\, dx = \pi.$$
3. It is
$$\sin x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}.$$
If we differentiate this, we get
$$\frac{d}{dx}\sin x = \sum_{n=0}^{\infty}(2n+1)\frac{x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = \cos x.$$
The other antiderivate is similar to show.
4. a)
$$\begin{aligned}\int_a^b f(x)\, dx &= \int_a^b \cos(x)\sin(x)\, dx \\ &= \sin(x)\sin(x)\Big|_a^b - \int_a^b \sin(x)\cos(x)\, dx.\end{aligned}$$
So we get
$$2\int_a^b \sin(x)\cos(x)\, dx = \sin(x)\sin(x)\Big|_a^b$$
which yields to
$$\int_a^b \sin(x)\cos(x)\, dx = \frac{1}{2}\sin(x)\sin(x)\Big|_a^b.$$
b)
$$\begin{aligned}\int_a^b g(x)\, dx &= \int_a^b xe^x\, dx \\ &= xe^x\Big|_a^b - \int_a^b e^x\, dx \\ &= xe^x\Big|_a^b - e^x\Big|_a^b \\ &= (x-1)e^x\Big|_a^b\end{aligned}$$