Mathematics with Computer Science

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Introductory Course — Fachbereich Mathematik
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Hints to Exercise, Unit 4

1.

	f_1	f_2	f_3	f_4	f_5	f_6
injective	–	×	×	×	×	–
surjective	–	–	×	×	–	×
bijective	–	–	×	×	–	–

2. (a) $f : \mathbb{N} \to \mathbb{N}, x \mapsto x + 1$
   (b) $f : \mathbb{N} \to \mathbb{N}, 1 \mapsto 1, x \mapsto x - 1$ for $x > 1$.

3. (a) $f(x) = \sqrt{x}$, $g(x) = x + 9$.
   (b) $f(x) = x^2$, $g(x) = x - 5$.
   (c) $f(x) = x + 2$, $g(x) = \sqrt{x}$.
   (d) $f(x) = \frac{1}{x}$, $g(x) = x - 1$.

4. (a) $(f \circ f)(x) = x^4$. The domain of $f \circ f$ is $\mathbb{R}$.
   $(g \circ f)(x) = x^2 - 3$. The domain of $g \circ f$ is $\mathbb{R}$.
   $(f \circ g)(x) = (x - 3)^2$. The domain of $f \circ g$ is $\mathbb{R}$.
   $(g \circ g)(x) = x - 6$. The domain of $g \circ g$ is $\mathbb{R}$.
   (b) $(f \circ g \circ h)(x) = \dfrac{(x + 3)^2}{(x + 3)^2 + 1}$.

5. Possible zeros of f are $\{\pm 1, \pm 2, \pm 3, \pm 6\}$. We try 1 and see, that $f(1) = 0$. Therefore we can write

$$f(x) = (x - 1)(x^2 - 5x + 6).$$

Applying the p/q-Formula yields to

$$\{x \in \mathbb{R} \ : \ f(x) = 0\} = \{1, 2, 3\}.$$

Possible zeros of g are $\{\pm 1\}$. We try 1 and see, that $f(1) = 0$. Therefore we can write

$$g(x) = (x - 1)(x^3 - 3x^2 + 3x - 1).$$

Again we try 1 and find

$$g(x) = (x - 1)^2(x^2 - 2x + 1).$$

Applying the p/q-Formula yields to

$$\{x \in \mathbb{R} \ : \ g(x) = 0\} = \{1\}.$$

Possible zeros of h are $\{\pm 1\}$. We try 1 and see, that $f(1) = 0$. Therefore we can write

$$h(x) = (x - 1)(x^3 + x^2 + x + 1).$$

We try -1 again and find

$$h(x) = (x - 1)(x + 1)(x^2 + 1).$$

Applying the p/q-Formula yields to

$$\{x \in \mathbb{R} \ : \ h(x) = 0\} = \{1, -1\}.$$

6. To determine the solutions of $f(x) = 1$, we solve

$$f(x) - 1 = 0.$$

This leads to

$$\{x \in \mathbb{R} \ : \ f(x) = 1\} = \{1, 2, -2\}.$$